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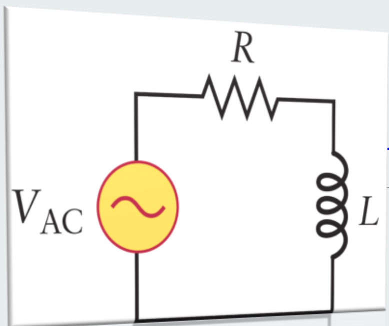
Magnetism and Alternating Current



Unit 4: Inductance

Lecture 18: RL Circuits

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Unit 4: Inductance

- 4.1 Self-Induction and Inductance
- 4.2 RL Circuits
- 4.3 Energy in a Magnetic Field
- 4.4 Mutual Inductance
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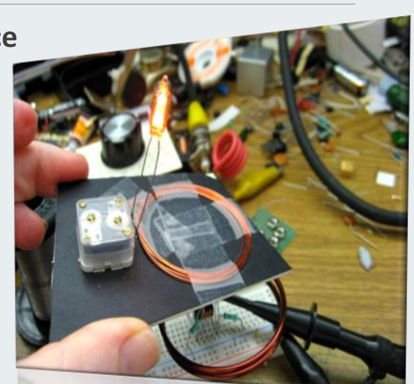
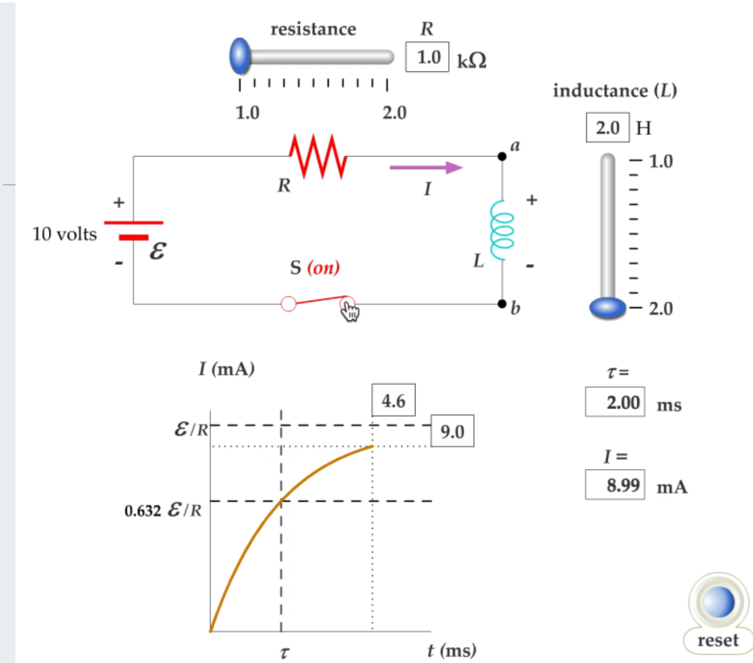
- 4.6 The RLC Circuit





RL Circuit, Introduction

A circuit element that has a large self-inductance is called an **inductor.**

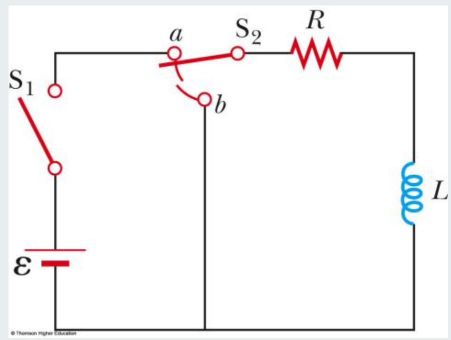
The circuit symbol is 

We assume the self-inductance of the rest of the circuit is negligible compared to the inductor.

- However, even without a coil, a circuit will have some self-inductance.

RL Circuit, Analysis

- An *RL* circuit contains an inductor and a resistor
- Assume S_2 is connected to *(a)*
- When switch S_1 is closed (at time t = 0), the current begins to increase.
- At the same time, a back emf is induced in the inductor that opposes the original increasing current.

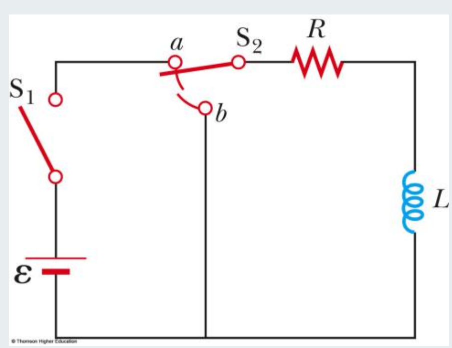


RL Circuit, Analysis, cont.

Applying Kirchhoff's loop rule to the previous circuit in the clockwise direction gives

$$\varepsilon - IR - L\frac{dI}{dt} = 0$$

Looking at the current as a function of time, we find



$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

To find this solution, we change variables for convenience, lets assume

$$x = (\mathcal{E}/R) - I \quad \longrightarrow \quad dx = -dI$$

$$x + \frac{L}{R}\frac{dx}{dt} = 0$$

Rearranging and integrating this last expression gives

$$\int_{x_0}^{x} \frac{dx}{x} = -\frac{R}{L}\int_0^t dt$$

where x_0 is the value of x at time t = 0.

$$\int_{x_0}^{x} \frac{dx}{x} = -\frac{R}{L}\int_0^t dt$$

$$\ln\frac{x}{x_0} = -\frac{R}{L}t$$

Taking the antilogarithm of this result gives

$$x = x_0 e^{-Rt/L} \qquad x = (\mathcal{E}/R) - I$$

Because I = 0 at t = 0, note from the definition of x that $x_0 = \varepsilon/R$. Hence, this last expression is equivalent to

$$\frac{\mathcal{E}}{R} - I = \frac{\mathcal{E}}{R}e^{-Rt/L}$$

$$I = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$$

Conclusion

$$I = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$$

- The inductor affects the current exponentially.
- The current does not instantly increase to its final equilibrium value.
- If there is no inductor, the exponential term goes to zero and the current would instantaneously reach its maximum value as expected.

RL Circuit, Time Constant

The expression for the current can also be expressed in terms of the time constant, τ, of the circuit

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$$

- where $\tau = L / R$

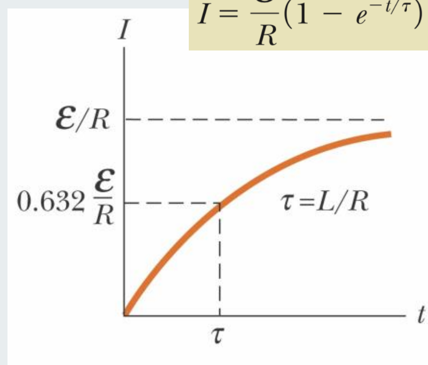
Physically, time constant t is the time required for the current to reach $(1 – e^{-1}) = 0.632 = 63.2\%$ of its maximum value ε/R.

The time constant is a useful parameter for comparing the time responses of various circuits.

RL Circuit, Current-Time Graph

- The equilibrium value of the current is $\mathcal{E}/R$ and is reached as t approaches infinity.
- The current initially increases very rapidly.
- The current then gradually approaches the equilibrium value.

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$$



RL Circuit, Current-Time Graph

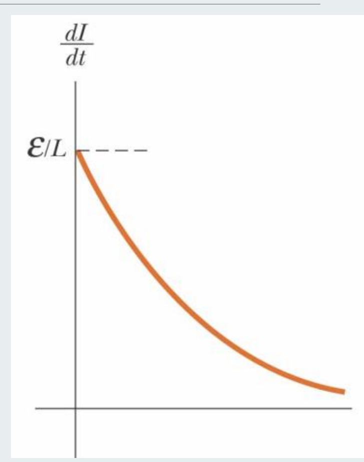
The time rate of change of the current by taking the first time derivative of

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$$

We get

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-t/\tau}$$

This result shows that the time rate of change of the current is a maximum (equal to ε/L) at $t = 0$ and falls off exponentially to zero as t approaches infinity.



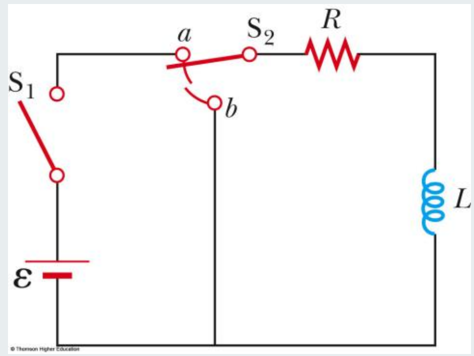
RL Circuit Without A Battery

Now set S_2 to position (b).

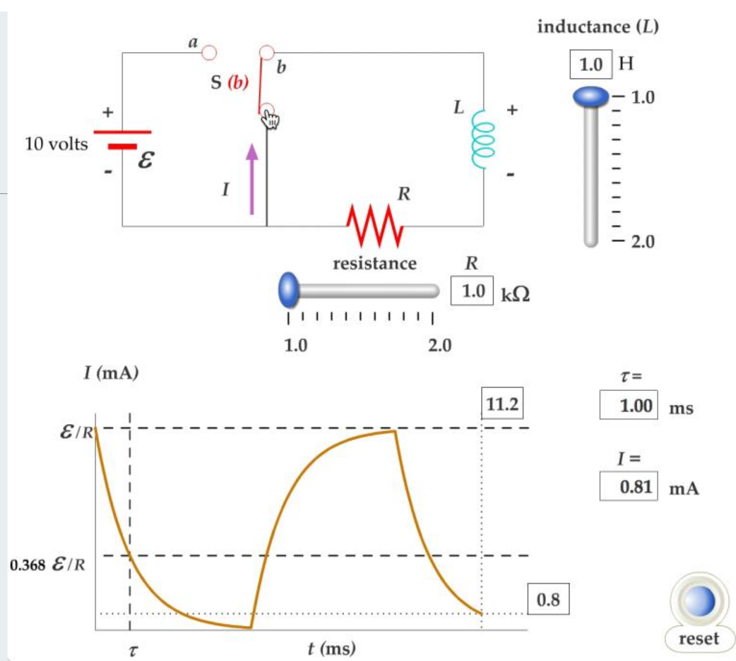
The circuit now contains just the right hand loop.

The battery has been eliminated.

The expression for the current becomes.

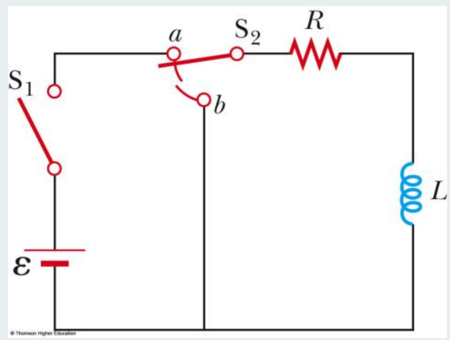


$$IR + L\frac{dI}{dt} = 0 \rightarrow I = \frac{\varepsilon}{R} e^{-t/\tau} = I_i e^{-t/\tau}$$



Example

Consider the circuit in the Figure. Suppose the circuit elements have the following values: ε = 12.0 V, *R* = 6.00 Ω, and *L* = 30.0 mH.



(A) Find the time constant of the circuit.

(B) Switch S_2 is at position *a,* and switch S_1 is thrown closed at *t* = 0. Calculate the current in the circuit at *t* = 2.00 ms.

(C) Compare the potential difference across the resistor with that across the inductor.

Solution (A) & (B)

(A) the time constant

$$\tau = \frac{L}{R} = \frac{30.0 \times 10^{-3}\ \text{H}}{6.00\ \Omega} = 5.00\ \text{ms}$$

(B) The current at *t*=2.00ms

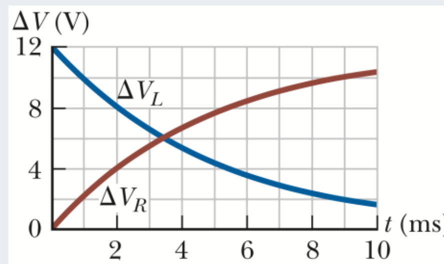
$$I = \frac{\varepsilon}{R}(1 - e^{-t/\tau}) = \frac{12.0\ \text{V}}{6.00\ \Omega}(1 - e^{-2.00\ \text{ms}/5.00\ \text{ms}}) = 2.00\ \text{A}\,(1 - e^{-0.400})$$

$$= 0.659\ \text{A}$$

Solution (C)

At the instant the switch is closed, there is no current and therefore no potential difference across the resistor.

At this instant, the battery voltage appears entirely across the inductor in the form of a back emf of 12.0 V as the inductor tries to maintain the zero-current condition. (The top end of the inductor in the Figure is at a higher electric potential than the bottom end.)



As time passes, the emf across the inductor decreases and the current in the resistor (and hence the voltage across it) increases as shown in the Figure. The sum of the two voltages at all times is 12.0 V.

Solve by your self

(1) A 510-turn solenoid has a radius of 8.00 mm and an over- all length of 14.0 cm. (a) What is its inductance? (b) If the solenoid is connected in series with a 2.50-V resistor and a battery, what is the time constant of the circuit?

(2) Consider the circuit shown in the Figure (a) When the switch is in position a, for what value of R will the circuit have a time constant of 15.0 ms? (b) What is the current in the inductor at the instant the switch is thrown to position b?