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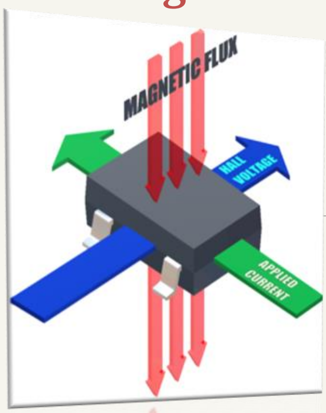
Magnetism and Alternating Current



Unit 1: Magnetic Fields

Lecture 6: The Hall Effect

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Unit 1: Magnetic Fields

1.1 Magnetic Fields and Forces.

1.2 Motion of a Charged Particle in a Uniform Magnetic Field.

1.3 Applications Involving Charged Particles Moving in a Magnetic Field.

1.4 Magnetic Force Acting on a Current-Carrying Conductor.

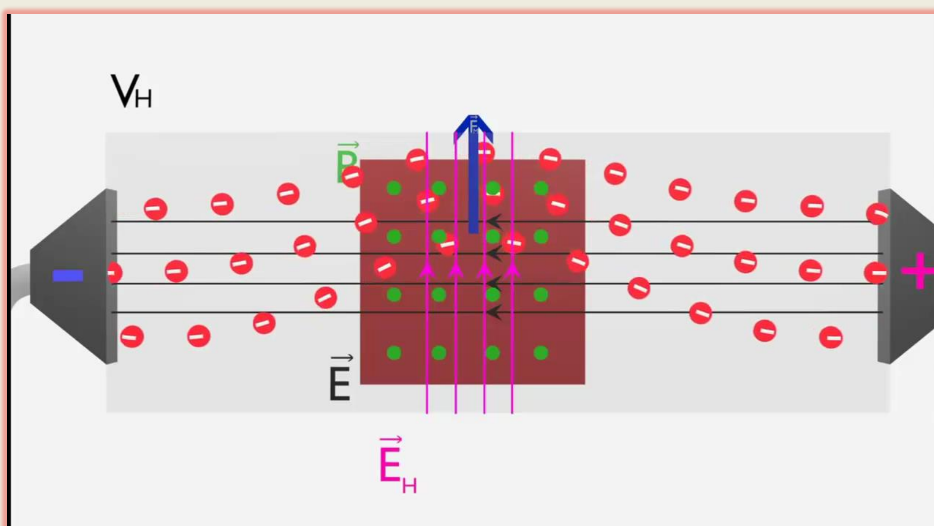
1.5 Torque on a Current Loop in a Uniform Magnetic Field.

1.6 The Hall Effect.

Hall effect

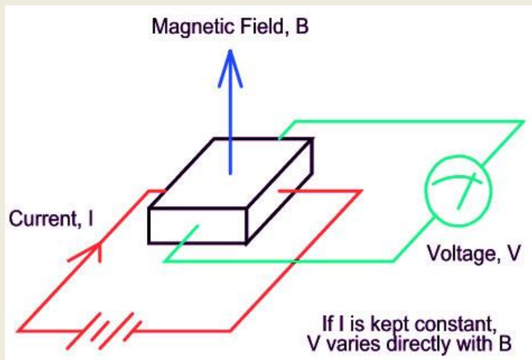
The Hall effect is the production of a voltage difference (the Hall voltage) across a current carrying conductor (in presence of magnetic field), perpendicular to both current and the magnetic field.

The Hall effect was discovered in 1879 by Edwin Hall while working on his doctoral degree at the Johns Hopkins University in Baltimore, Maryland, USA.



Observing the Hall effect

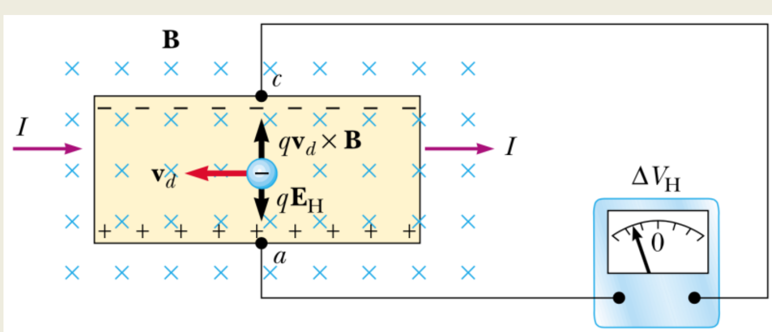
The arrangement for observing the Hall effect consists of a flat conductor carrying a current I in the x direction. A uniform magnetic field B is applied in the y direction.



If the charge carriers are electrons moving in the negative x direction with a drift velocity $\mathbf{v}_d$, they experience an upward magnetic force

$$\vec{F}_B = q\vec{v}_d \times \vec{B}$$

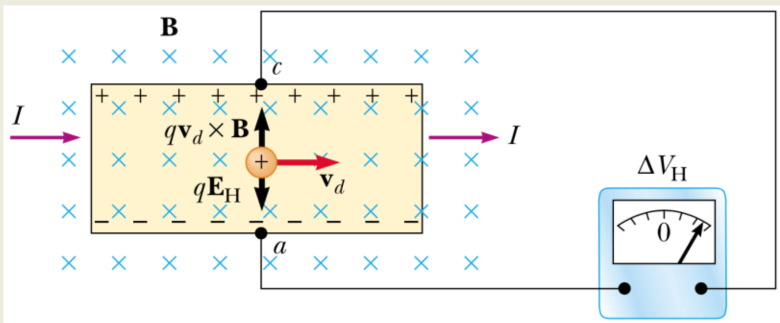
are deflected upward, and accumulate at the upper edge of the flat conductor, leaving an excess of positive charge at the lower edge.



This accumulation of charge at the edges establishes an electric field in the conductor and increases until the electric force balances the magnetic force acting on the carriers.

When this equilibrium condition is reached, deflection stops.

A sensitive voltmeter connected across the sample as shown in the Figure can measure the potential difference, known as the Hall voltage ΔV_H , generated across the conductor.



Deriving an expression for the Hall voltage

The magnetic force exerted on the carriers has magnitude qv_dB. In equilibrium, this force is balanced by the electric force qE_H , (E_H is the Hall field).

$$qv_dB = qE_H$$

$$E_H = v_dB$$

If d is the width of the conductor, the **Hall voltage** is

$$\Delta V_H = E_Hd = v_dBd$$

Therefore, the measured Hall voltage gives a value for the drift speed of the charge carriers if d and B are known

The charge-carrier density n

We can obtain the charge-carrier density n by measuring the current in the sample. From Equation of the drift velocity and the current, we can express the drift speed as

$$v_d = \frac{I}{nqA}$$

where A is the cross-sectional area of the conductor.

$$\therefore \ \Delta V_H = v_d B d$$

$$\Delta V_H = \frac{IBd}{nqA}$$

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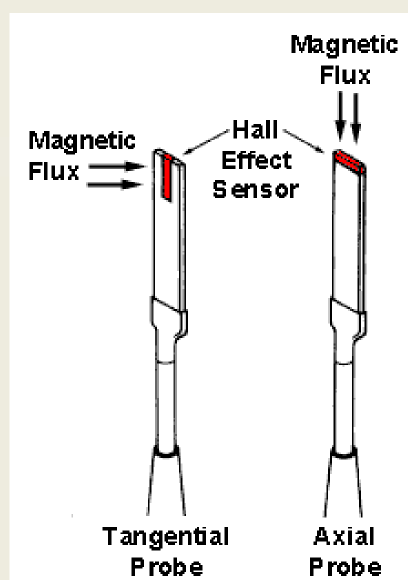
Because A = td, where t is the thickness of the conductor, we can also express

$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$

where $R_H = 1/nq$ is called the Hall coefficient.

This relationship shows that a properly calibrated conductor can be used to measure the magnitude of an unknown magnetic field.



Hall Effect Measurement Experiment

Example 1

$$\Delta V_H = \frac{IB}{nqt}$$

A rectangular copper strip 1.5 cm wide and 0.10 cm thick carries a current of 5.0 A. Find the Hall voltage for a 1.2-T magnetic field applied in a direction perpendicular to the strip.

Solution

Assuming one electron per atom is available for conduction, find the charge-carrier density in terms of the molar mass M and density ρ of copper:

$$n = \frac{N_A}{Volume} = \frac{N_A \rho}{M}$$

$$n = \frac{N_A \rho}{M}$$

$$\Delta V_H = \frac{IB}{nqt} = \frac{MIB}{N_A \rho q t}$$

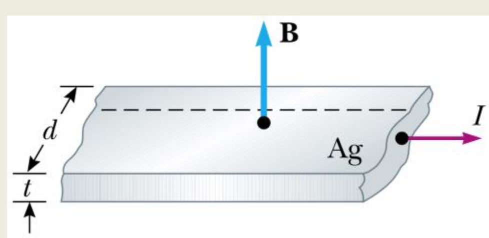
$$\Delta V_H = \frac{(0.063\,5\ \text{kg/mol})(5.0\ \text{A})(1.2\ \text{T})}{(6.02 \times 10^{23}\ \text{mol}^{-1})(8\,920\ \text{kg/m}^3)(1.60 \times 10^{-19}\ \text{C})(0.001\,0\ \text{m})}$$

$$= 0.44\ \mu\text{V}$$

Such an extremely small Hall voltage is expected in good conductors. What if the strip has the same dimensions but is made of a semiconductor? Will the Hall voltage be smaller or larger?

Example 2

A flat ribbon of silver having a thickness t = 0.200 mm is used in a Hall-effect measurement of a uniform magnetic field perpendicular to the ribbon, as shown in the Figure. The Hall coefficient for silver is R_H = 0.840 x 10^{10} m^3/C. **(a)** What is the density of charge carriers in silver? **(b)** If a current I = 20.0 A produces a Hall voltage V_H = 15.0 V, what is the magnitude of the applied magnetic field?



Solution (A)

(A) charge carriers in silver

$R_H = 1/nq$

$$n = \frac{1}{qR_H}$$

$$= \frac{1}{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(0.840 \times 10^{-10}\ \text{m}^3/\text{C}\right)}$$

$$= \boxed{7.44 \times 10^{28}\ \text{m}^{-3}}$$

Solution (B)

(B) magnitude of the applied magnetic field

$$\text{D}V_H = \frac{IB}{nqt} \quad \longrightarrow \quad B = \frac{nqt(\Delta V_H)}{I}$$

$$= \frac{\left(7.44 \times 10^{28}\ \text{m}^{-3}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)\left(0.200 \times 10^{-3}\ \text{m}\right)\left(15.0 \times 10^{-6}\ \text{V}\right)}{20.0\ \text{A}}$$

$$= \boxed{1.79\ \text{T}}$$

Solve by your self

1. A Hall-effect probe operates with a 120-mA current. When the probe is placed in a uniform magnetic field of magnitude 0.0800T, it produces a Hall voltage of 0.700V. (a) When it is measuring an unknown magnetic field, the Hall voltage is 0.330 μV. What is the magnitude of the unknown field? (b) The thickness of the probe in the direction of B is 2.00 mm. Find the density of the charge carriers, each of which has charge of magnitude *e*.

2. In an experiment that is designed to measure the Earth's magnetic field using the Hall effect, a copper bar 0.500 cm thick is positioned along an east–west direction. If a current of 8.00 A in the conductor results in a Hall voltage of 5.10×10^{12} V, what is the magnitude of the Earth's magnetic field? (Assume that $n = 8.49 \times 10^{28}$ electrons/m^3 and that the plane of the bar is rotated to be perpendicular to the direction of **B**.)