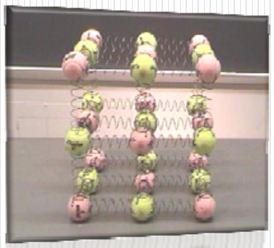
Selected Topics in Physics for Medical Sciences Students



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Unit 3
Elastic Properties of Materials
Lecture 9
Hook's Law

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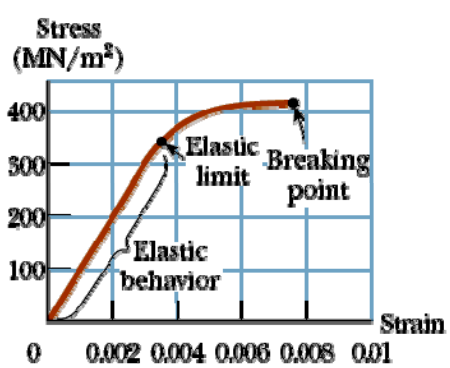
Young's Modulus

- In the linear region of the Strain vs. Stress diagram, the slope equals the stress-to-strain ratio and is called Young' Modulus or Elastic Modulus (E) of the material

$$\text{Elastic modulus} \equiv \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{\sigma}{\varepsilon}$$



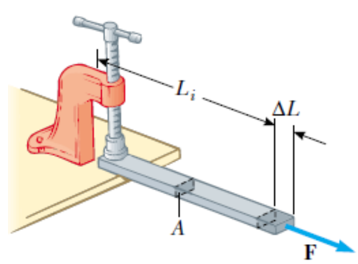
Deformation types and define an elastic modulus

1. **Young's modulus,** which measures the resistance of a solid to a change in its length.
2. **Shear modulus,** which measures the resistance to motion of the planes within a solid parallel to each other.
3. **Bulk modulus,** which measures the resistance of solids or liquids to changes in their volume.

Young's Modulus: Elasticity in Length

$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L / L_0}$$

Young's modulus



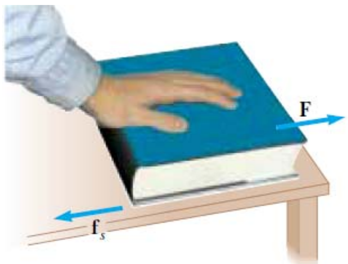
- Young's modulus is typically used to characterize a rod or wire stressed under either tension or compression.

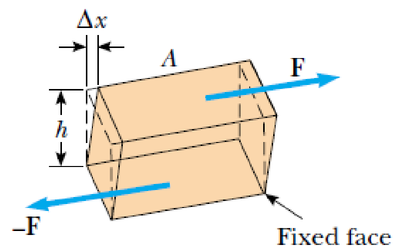
Shear Modulus: Elasticity of Shape

$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta x/h}$$

shear modulus

Where Δx is the horizontal distance that the sheared face moves and h is the height of the object.



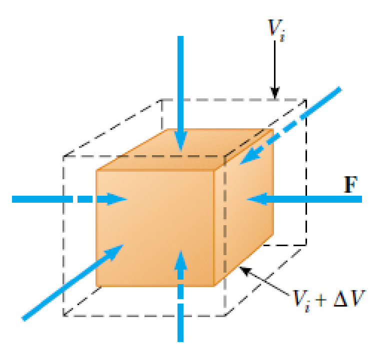


Bulk Modulus: Volume Elasticity

$$Y = \frac{\sigma}{\varepsilon} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i}$$

bulk modulus

- A negative sign is inserted in this defining equation so that Y *is a positive number. This* maneuver is necessary because an increase in pressure (positive ΔP) *causes a decrease* in volume (negative ΔV) *and vice versa*.



Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Example

How much pressure is needed to compress the volume of an iron block by 0.10 percent? Express answer in N/m^2, and compare it to atmospheric pressure ($1.0 \times 10^5\ N/m^2$).

Solution:

Y for iron = $90 \times 10^{10}\ N/m^2$

$$Y = -\frac{\Delta P}{\Delta V / V_i}$$

$$\frac{\Delta V}{V_i} = \frac{0.1}{100} = 10^{-3}$$

$$90 \times 10^{10} = -\frac{\Delta P}{10^{-3}}$$

$$\therefore \Delta P = 90 \times 10^{6}\ N/m^2$$

Hook's Law

- The linear stress-strain region in the diagram is also called **Hook's Law** region. In this region, since the stress is linearly related to the strain. The force is linearly related to the elongation.

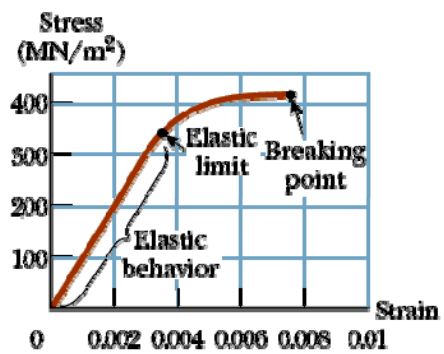
$$Y = \frac{\sigma}{\varepsilon}$$

$$\sigma = Y\varepsilon$$

$$\varepsilon = \frac{\Delta l}{l_o}$$

$$\sigma = \frac{F}{A}$$

$$\frac{F}{A} = Y\frac{\Delta l}{l_o}$$



$$\frac{F}{A} = Y\frac{\Delta l}{l_o}$$

- Thus, in tension or compression the force on an object is proportional to its elongation.

$$F = k\Delta l$$ Hook's law

- Where **k** is the spring constant

$$k = \frac{YA}{l_o}$$

ينطبق قانون هوك على المواد التي في منطقة التغير الخطي بين الإجهاد stress والانفعال strain وتكون قيمة k كبيرة كلما زاد مساحة المقطع وقل طوله.

Example 10-2

In example 10-1, a $10^8 N/m^2$ stress produces a strain of $5x10^{-4}$.

What is Young's modulus for this bar?

Solution

$$Y = \frac{\sigma}{\varepsilon} = \frac{10^8}{5\times10^{-4}} = 20\times10^{10}\,N/m^2$$

Example

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is 1.0 $x10^5$ N/m^2 (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is 2.0 x 10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

Solution

- From the definition of bulk modulus, we have

$$Y = -\frac{\Delta P}{\Delta V / V_i}$$

$$\Delta V = -\frac{V_i \Delta P}{Y}$$

$$\Delta V = -\frac{(0.50m^2)(2.0\times10^7 N/m^2 - 1.0\times10^5 N/m^2)}{6.1\times10^{10} N/m^2}$$

$$\Delta V = -1.6\times10^{-4} m^3$$

The negative sign indicates that the volume of the sphere decreases.

Example

A vertical steel girder (عارضة خشبية) with a cross-sectional area of 0.15 m² has a 1550 kg sign hanging from its end. (Ignore the mass of the girder itself.)

- **(a) What is the stress within the girder?**
- **(b) What is the strain on the girder?**
- **(c) If the girder is 9.50 m long, how much is it lengthened?**

Solution

- A = 0.15 m^2, m = 1550 kg, l_o = 9.5m
- F = mg = 1550 x 9.8 = 15200 N

$$\sigma = \frac{F}{A} = \frac{mg}{A} = \frac{15200}{0.15} = 1.0 \times 10^5 \, N/m^2$$

$$Y = \frac{\sigma}{\varepsilon}$$

$$\therefore \varepsilon = \frac{\sigma}{Y} = \frac{1.0 \times 10^5}{200 \times 10^9} = 5.0 \times 10^{-7}$$

$$\Delta l = \varepsilon \times l_o = 4.8 \times 10^{-6} \, m$$

Example

A 15 cm long animal tendon (وتر) was found to stretch 3.7 mm by a force of 13.4 N. The tendon was approximately round with an average diameter of 8.5 mm.

- **Calculate the elastic modulus of this tendon.**

Solution

- Δl = 3.7 mm, l_0 = 15 cm, 2r = 8.5 mm and **F = 13.4 N**
- Δl = 3.7x10^{-3} m, l_0 = 0.15 m, r = 4.25x10^{-3} m
- The cross sectional area A = πr^2 = 5.7x10^{-5} m^2

$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta l / l_0} = 9.5 \times 10^6 \, N/m$$

Problem

- **A nylon tennis string on a racquet is under a tension of 250 N. If its diameter is 1.00 mm, by how much is it lengthened from its un-tensioned length?**