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General Physics I

Mechanics: Principles & Applications

Lecture (2): Units, Dimensions and Vectors

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VECTOR AND SCALAR

Scalars: One that can be described by a single number (along with the unit)

- Water freezes at a temperature of 0^o C or 32^o F
- The mass of a book is 198.2 g
- The length of room is 5 m
- The car kinetic energy was 0.345 J

Vectors: A quantity that deals with magnitude and direction is called a vector quantity.

- The wind had a velocity of 25 km/h from the North
- The momentum was 1.234 kg m/s to the left

Textbooks use either A or $\vec{A}$

Vector and scalar quantities

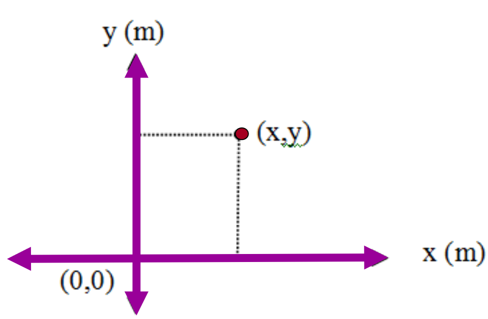
Vector Quantity	Scalar Quantity
Displacement	Length
Velocity	Mass
Force	Speed
Acceleration	Power
Field	Energy
Momentum	Work

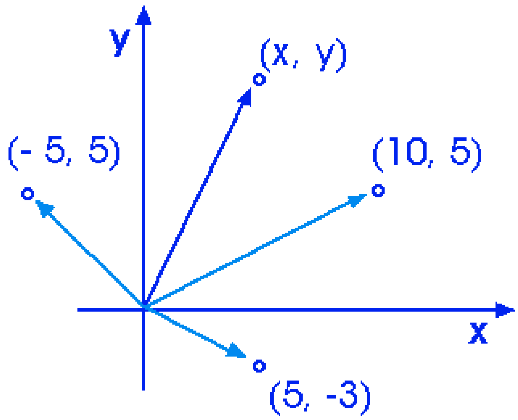
COORDINATE SYSTEM (1)

(1) The rectangular coordinates

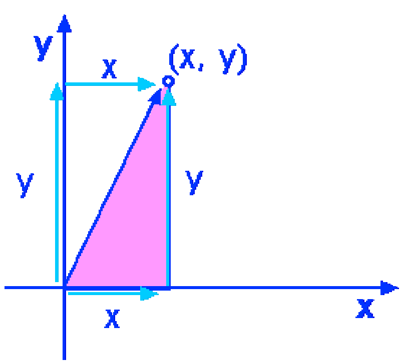
This coordinate system is consist of a fixed reference point (0,0) which called the origin.

A set of axis with appropriate scale and label.





The x- and y-coordinates may be either positive or negative



We can call this x-component, a vector along the x-direction of length x, and indicate that it is a vector by x.

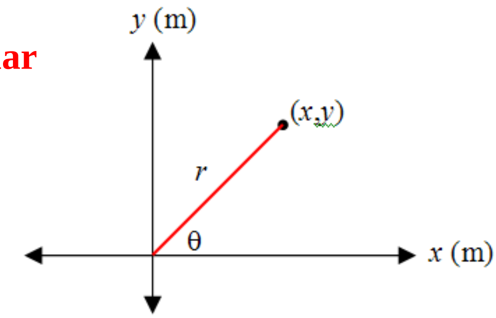
Likewise, we can call this y-component, a vector along the y-direction of length y, and indicate that it is a vector by y .

r = x + y

Coordinate system (2)

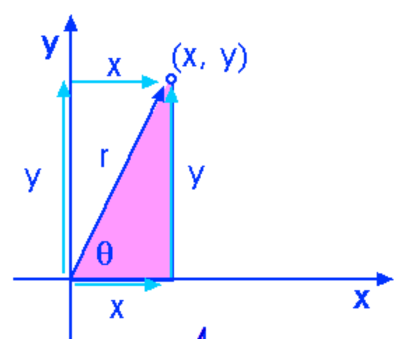
(2) The polar coordinates

Sometimes it is more convenient to use the polar coordinate system (r,θ), where r is the distance from the origin to the point of rectangular coordinate (x,y), and θ is the angle between r and the x axis.



THE RELATION BETWEEN COORDINATES

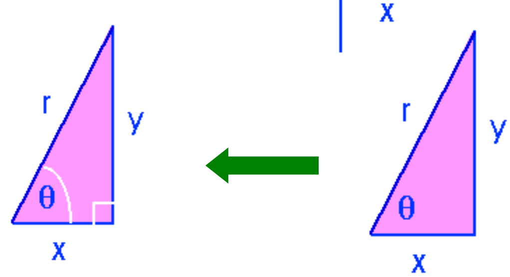
The relation between the rectangular coordinates (x,y) and the polar coordinates (r,θ) is shown in Figure,

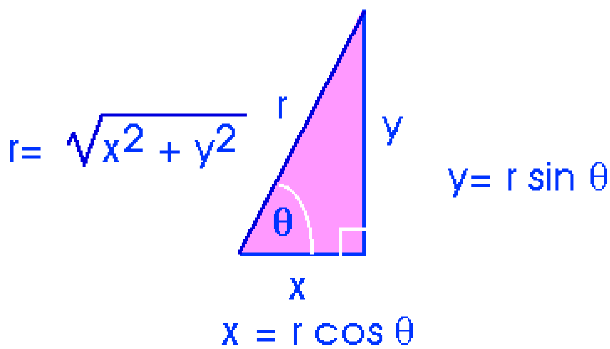


$\sin\theta = y/r$

$\cos\theta = x/r$

$\tan\theta = y/x$





It is common practice to measure the angle from the positive x-axis and to measure it positive for a counterclockwise direction.

Example 1

The example shown below might be for an angle of θ = 53º .Then, if r = 10, the components will be

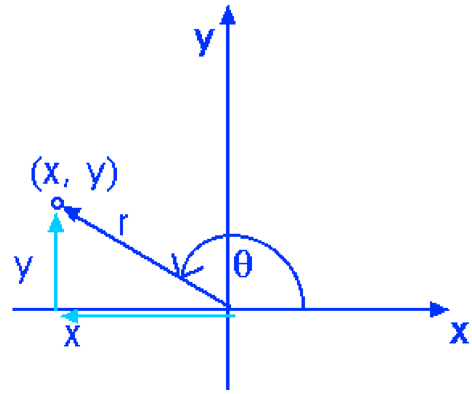
x = r cos θ= (0.6) (10) =6

y = r sin θ = (0.8) (10) = 8

Of course, angle θ does not need to be limited to the first quadrant. for θ = 150º, r = 10 for this numerical example. For that case,

x = r cos θ = (10)(cos 150º) = (10)(-0.866)= -8.66

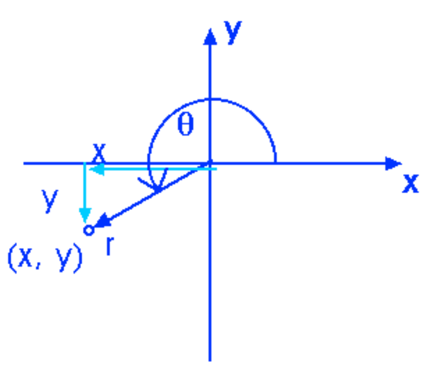
y = r sin θ = (10)(sin 150º) = (10)(0.500) = 5.00



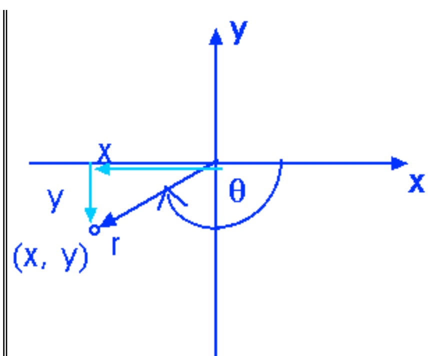
notice the signs and compare them with the diagram.

x = - 8.66 is located to the left and y = 5.00 is located up .

من الممكن ان نقيس الزاوية لتحديد الاتجاه من محور x مع عقارب عكس عقارب الساعة أو مع عقارب الساعة ولكن يجب ان نميز ذلك باشارة الزاوية.



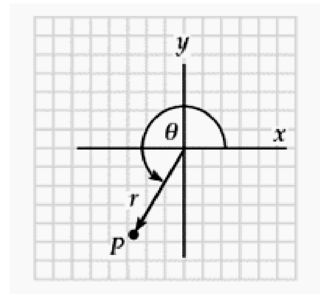
r = 10, θ = 210º



r = 10, θ = -150º

Example 2

The polar coordinates of a point are r = 5.5m and θ=240º. What are the Cartesian coordinates of this point?



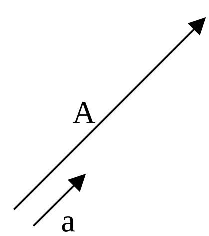
- **Solution**

$x = r \cos \theta = 5.5 \times \cos 240^{o} = -2.75$ m

$y = r \sin \theta = 5.5 \times \sin 240^{o} = -4.76$ m

The unit vector

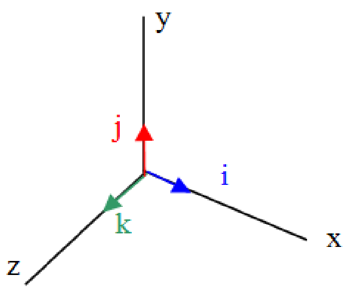
A unit vector is a vector having a magnitude of unity and its used to describe a direction in space.



المتجه ***A*** يمكن تمثيله بمقدار المتجه ضرب متجه الوحدة ***a*** كالتالي:

$$\mathbf{A} = \boldsymbol{a}\, A$$

كذلك يمكن تمثيل متجهات وحدة (i, j, k) لمحاور الإسناد المتعامدة (x, y, z) كما في الشكل التالي:-



i ≡ a unit vector along the *x*-axis

j ≡ a unit vector along the *y*-axis

k ≡ a unit vector along the *z*-axis

Instead of explicitly writing $A_x = 5, A_y = 0$; $B_x = 5, B_y = 5$; $C_x = -10, C_y = 0$; and $D_x = -5, D_y = 5$, we can write this same information in a different form. We can write

$$A = 5\,i + 0j$$

$$B = 5i + 5j$$

$$C = -10i + 0j$$

$$D = -5i + 5j$$

$$R = A + B + C + D$$

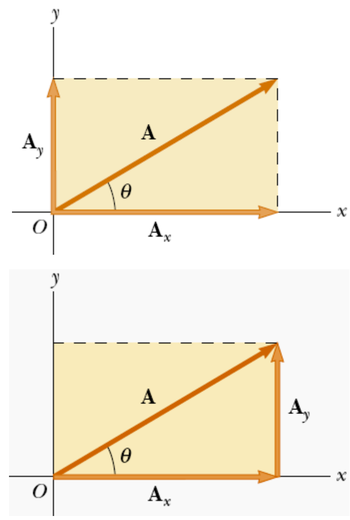
$$R = (5\,i + 0j)+(5i + 5j)+(-10i + 0j)+(-5i + 5j)$$

$$R = (5+5-10-5)i + (0+5+0+5)j$$

$$R = -5i + 10j$$

Components of a vector

Any vector lying in *xy* plane can be resolved into two components one in the *x*-direction and the other in the *y*-direction as shown in Figure



$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

عند التعامل مع عدة متجهات فإننا نحتاج إلى تحليل كل متجه إلى مركباته بالنسبة إلى محاور الإسناد (x,y) مما يسهل إيجاد المحصلة بدلاً من استخدام الطريقة البيانية لإيجاد المحصلة.

The magnitude of the vector **A**

$$A = \sqrt{A_x^2 + A_y^2}$$

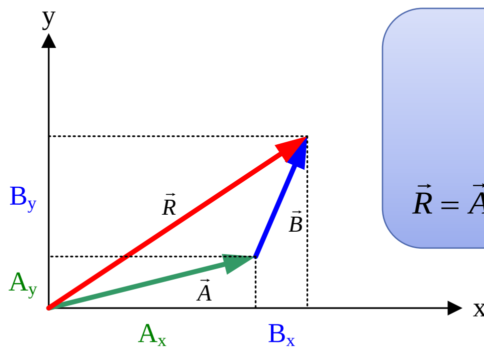
The direction of the vector to the *x*-axis

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

A vector **A** lying in the *xy* plane, having rectangular components A_x and A_y can be expressed in a unit vector notation

$$\mathbf{A} = A_x\mathrm{i} + A_y\mathrm{j}$$

ملاحظة: يمكن استخدام طريقة تحليل المتجهات في جمع متجهين A و B كما في الشكل التالي:



$$\vec{A} = A_x i + A_y j$$

$$\vec{B} = B_x i + B_y j$$

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)i + (A_y + B_y)j$$

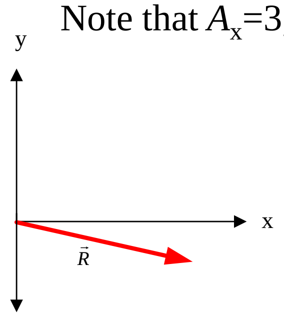
Example 3

Find the sum of two vectors A and B and given by

$$\vec{A} = 3i + 4j \text{ and } \vec{B} = 2i - 5j$$

Solutions

Note that A_x=3, A_y=4, B_x=2, and B_y=-5



$$\vec{R} = \vec{A} + \vec{B} = (3+2)i + (4-5)j = 5i - j$$

The magnitude of vector R is

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{25+1} = \sqrt{26} = 5.1$$

The direction of R with respect to *x*-axis is

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{-1}{5} = -11^o$$

Example 4

Two vectors are given by $\vec{A} = 3i - 2j$ and $\vec{B} = -i - 4j$. Calculate (a) $\vec{A}+\vec{B}$, (b) $\vec{A}-\vec{B}$, (c) $|\vec{A}+\vec{B}|$, (d) $|\vec{A}-\vec{B}|$, and (e) the direction of $\vec{A}+\vec{B}$ and $|\vec{A}-\vec{B}|$.

Solutions

(a) $\vec{A}+\vec{B} = (3i-2j)+(-i-4j) = 2i-6j$

(b) $\vec{A}-\vec{B} = (3i-2j)-(-i-4j) = 4i+2j$

(c) $|\vec{A}+\vec{B}| = \sqrt{2^2+(-6)^2} = 6.32$

(d) $|\vec{A}-\vec{B}| = \sqrt{4^2+2^2} = 4.47$

(e) For $\vec{A}+\vec{B}$, $\theta = \tan^{-1}(-6/2) = -71.6^\circ = 288^\circ$

For $\vec{A}-\vec{B}$, $\theta = \tan^{-1}(2/4) = 26.6^\circ$

Example 5

A particle moves from a point in the *xy* plane having rectangular coordinates (-3,-5)m to a point with coordinates (-1,8)m. (a) Write vector expressions for the position vectors in unit vector form for these two points. (b) What is the displacement vector?

Solution

(a)

$$\vec{R}_1 = x_1 i + y_1 j = (-3i - 5j)m$$

$$\vec{R}_2 = x_2 i + y_2 j = (-i + 8j)m$$

$$\Delta\vec{R} = \vec{R}_2 - \vec{R}_1$$

(b) Displacement =

$$\Delta\vec{R} = (x_2 - x_1)i + (y_2 - y_1)j = -i - (-3i) + 8j - (-5j) = (2i + 13j)m$$

EXAMPLE 6

A vector A has a negative *x* component 3 units in length and positive *y* component 2 units in length.

(a) Determine an expression for A in unit vector notation.

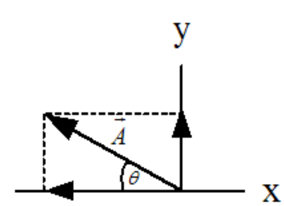
(b) Determine the magnitude and direction of A.

(c) What vector B when added to A gives a resultant vector with no *x* component and negative *y* component 4 units in length?

Solution

A_x = -3 units & A_y = 2 units

(a) **A= A_xi+A_yj=-3i+2j units**



(b) $|\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3)^2 + (2)^2} = 3.61\ units$

θ = $\tan^{-1}(2/-3)$ = 33.7° (relative to the –x axis)

(c) What vector B when added to A gives a resultant vector with no *x* component and negative *y* component 4 units in length?

$R_x = 0$ & $R_y = -4$; since $\vec{R} = \vec{A} + \vec{B}$, $\vec{B} = \vec{R} - \vec{A}$

$B_x = R_x - A_x = 0 - (-3) = 3$

$B_y = R_y - A_y = -4 - 2 = -6$

Therefore

$$\vec{B} = B_x i + B_y j = (3i - 6j)\ units$$