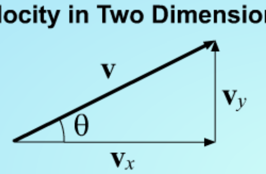
Two-dimensional kinematics is similar to one-dimensional kinematics, but we must now use full vector notation rather than positive and negative signs to indicate the direction of motion.

One Dimension	***Two Dimensions***
Position: ***x***	**Position vector:** ***r***
Displacement: ***Δx***	**Displacement vector:** ***Δ r***
Velocity: displacement per unit time. Sign is equal to the sign of the displacement *Δ x*	**Velocity vector: change in the position vector per unit time. The direction is equal to the direction of the displacement vector *Δ r*.**
Acceleration: change in velocity *Δ v* per unit time. Sign is equal to the sign of the velocity difference *Δ v*.	**Acceleration vector: change in the velocity vector per unit time. The direction is equal to the direction of the velocity difference vector *Δ v*.**

Motion in Two Dimensions

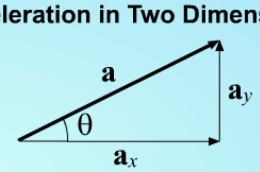
The motion in two dimensions can be modeled as two *independent* motions in each of the two perpendicular directions associated with the *x* and *y* axes. That is, any influence in the *y* direction does not affect the motion in the *x* direction and vice versa.

Velocity in Two Dimensions



$$v_x = v\cos\theta$$
$$v_y = v\sin\theta$$
$$|v| = \sqrt{v_x^2 + v_y^2}$$
$$\tan\theta = \frac{v_y}{v_x}$$

Acceleration in Two Dimensions



$$a_x = a\cos\theta$$
$$a_y = a\sin\theta$$
$$|a| = \sqrt{a_x^2 + a_y^2}$$
$$\tan\theta = \frac{a_y}{a_x}$$

Kinematics Equations with Constant Acceleration

When the acceleration is constant then we can substitute

$$v_x = v_{xo} + a_x t \qquad v_y = v_{yo} + a_y t$$

In

$$\vec{v} = v_x i + v_y j$$

We get

$$v = (v_{xo} + a_x t)i + (v_{yo} + a_y t)j$$

$$v = (v_{xo} i + v_{yo} j) + (a_x i + a_y j)t$$

then

$$v = v_o + at$$

من المعادلة نستنتج أن سرعة جسم عند زمن محدد t يساوى الجمع الاتجاهى للسرعة الابتدائية والسرعة الناتجة من العجلة المنتظمة.

Kinematics Equations with Constant Acceleration

Since our particle moves in two dimension x and y with constant acceleration then

$$x = x_o + v_{xo} t + \tfrac{1}{2} a_x t^2 \qquad y = y_o + v_{yo} t + \tfrac{1}{2} a_y t^2$$

but

$$r = x_i + y_j$$

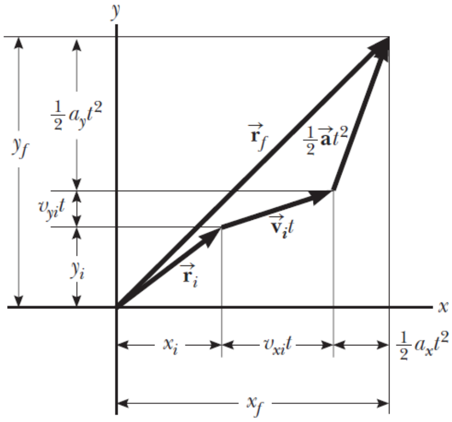
$$r = \left(x_o + v_{xo} t + \frac{1}{2} a_x t^2\right) i + \left(y_o + v_{yo} t + \frac{1}{2} a_y t^2\right) j$$

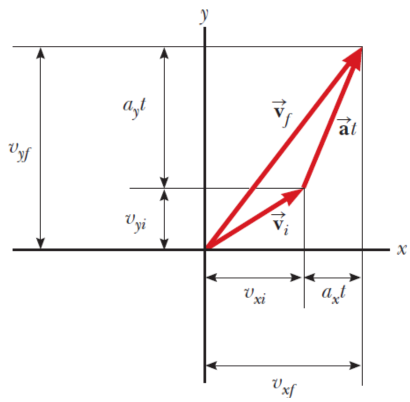
$$r = (x_o i + y_o j) + (v_{xo} i + v_{yo} j)t + \frac{1}{2}(a_x i + a_y j)t^2$$

$$r = r_o + v_o t + \frac{1}{2} a t^2$$

من المعادلة نستنتج أن متجه الإزاحة r-r_o هو عبارة عن الجمع الإتجاهى لمتجه الإزاحة الناتج عن السرعة الابتدائية $v_o t$ والإزاحة الناتجة عن العجلة المنتظمة $\frac{1}{2} at^2$.

Vector representations and components of the velocity and the position of a particle moving with a constant acceleration $\vec{a}$.





$$r = r_o + v_o t + \frac{1}{2}at^2$$

$$v = v_o + at$$

Kinematics Equations with Constant Acceleration

One Dimension

$$x = x_0 + v_0t + \frac{1}{2}at^2 \qquad v = v_0 + at$$

Two Dimensions

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0t + \frac{1}{2}\mathbf{a}t^2 \qquad \mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2 \qquad v_x = v_{0x} + a_xt$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2 \qquad v_y = v_{0y} + a_yt$$