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General Physics I

Mechanics: Principles & Applications

Lecture (5): Motion Kinematics
One-Dimensional Motion with Constant Acceleration

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- The average and Instantaneous velocity
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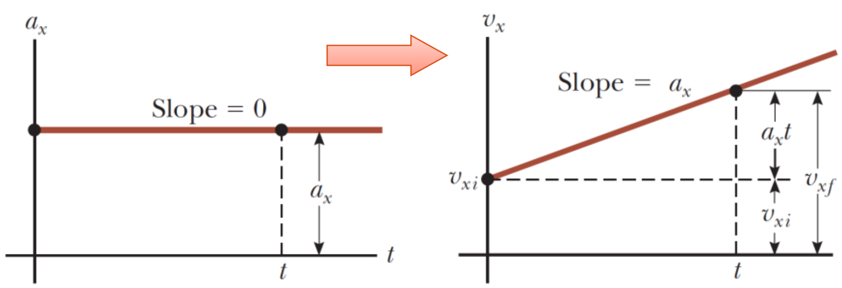
One-dimensional Motion With *Constant Acceleration*

سندرس الآن الحركة في بعد واحد وذلك فقط عندما يكون فيها التسارع ثابتا constant acceleration. وفي هذه الحالة يكون التسارع اللحظي Instantaneous acceleration يساوى متوسط التسارع Average acceleration.

Instantaneous acceleration = Average acceleration



نتيجة لذلك فإن السرعة إما أن تتزايد أو تتناقص بمعدلات متساوية خلال الحركة. ويعبر عن ذلك رياضياً على النحو التالي:-

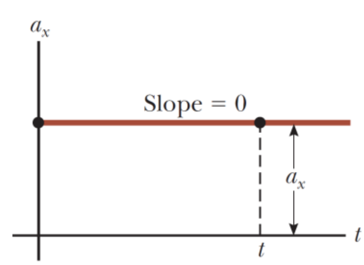
Kinematic Equations at Constant Acceleration

$$a = a_{ave} = \frac{v - v_o}{t - t_o}$$

Let $t_o = 0$ then the acceleration

$$a = \frac{v - v_o}{t}$$

$$v = v_0 + at$$



من هذه المعادلة يمكن إيجاد السرعة ***v*** عند أي زمن ***t*** إذا عرفنا السرعة الابتدائية ***v***v_o والتسارع الثابت ***a*** الذي يتحرك به الجسم. وإذا كان التسارع يساوي صفراً فإن السرعة لا تعتمد على الزمن، وهذا يعني أن السرعة النهائية تساوي السرعة الابتدائية. لاحظ أيضاً أن كل حد من حدود المعادلة السابقة له بعد سرعة (m/s).

Kinematic Equations at Constant Acceleration

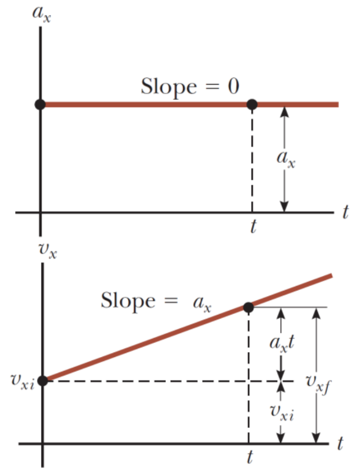
Since the velocity varies linearly (خطي) with time we can express the average velocity as

$$v_{ave} = \frac{v + v_{\circ}}{2}$$

To find the displacement Δ*x* (*x*-*x*$_o$) as a function of time

$$\Delta x = v_{ave}\Delta t = \left(\frac{v + v_{\circ}}{2}\right)t$$

$$x = x_o + \frac{1}{2}(v + v_{\circ})t$$



This equation provides the final position of the particle at time *t* in terms of the initial and final velocities.

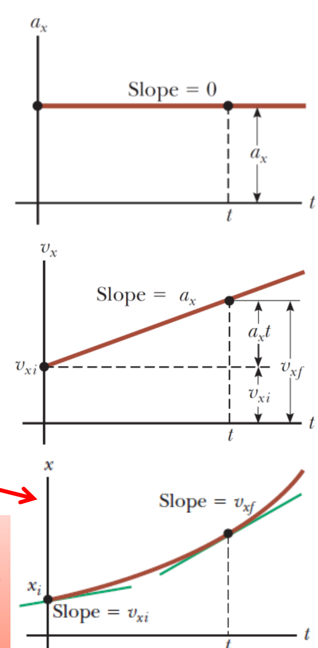
We can obtain another useful expression for the position of a particle under constant acceleration

$$x = x_o + \frac{1}{2}(v + v_{\circ})t$$

$$v = v_0 + at$$

$$x = x_o + \frac{1}{2}(v_o + at + v_{\circ})t$$

$$x = x_o + v_o t + \tfrac{1}{2}at^2$$



This equation provides the final position of the particle at time *t* in terms of the initial position, the initial velocity, and the constant acceleration.

We can obtain an expression for the final velocity that does not contain time as a variable

$$x = x_o + \frac{1}{2}(v + v_o)t$$

$$v = v_0 + at \longrightarrow \therefore t = \frac{v - v_o}{a}$$

$$x = x_o + \frac{1}{2}(v + v_o)\frac{v - v_o}{a} \longrightarrow x = x_o + \frac{v^2 - v_o{}^2}{2a}$$

$$v^2 = v_0{}^2 + 2a(x - x_o)$$

This equation provides the final velocity in terms of the initial velocity, the constant acceleration, and the position of the particle

Kinematic Equations at Constant Acceleration

$v = v_0 + at$	**Velocity as a function of time**
$x = x_o + \frac{1}{2}(v + v_o)t$	**Position as a function of velocity and time**
$x = x_o + v_o t + \frac{1}{2}at^2$	**Position as a function of time**
$v^2 = v_0{}^2 + 2a(x - x_o)$	**Velocity as a function of position**

General Problem solving Strategy

- Read Problem
- Draw Diagram
- Label physical quantities
- Choose Equation (s)
- Solve Equation (s)
- Substitute known values
- Chcck Answer

Example 1

A body moving with uniform acceleration has a velocity of 12cm/s when its *x* coordinate is 3cm. If its *x* coordinate 2s later is -5cm, what is the magnitude of its acceleration?

Solution

$$x = x_o + v_o t + \tfrac{1}{2} a t^2$$

$$-5 = 3 + 12\times2 + 0.5\, a\, (2)^2$$

$$a = -16 \text{ cm/s}^2$$

Example 2

A car moving at constant speed of 30m/s suddenly stalls at the bottom of a hill. The car undergoes a constant acceleration of -2m/s^2 while ascending the hill.

1. Write equations for the position and the velocity as a function of time, taking x=0 at the bottom of the hill where v_o = 30m/s.

Solution

$$x = x_o + v_o t + \tfrac{1}{2} a t^2$$

$$x = 0 + 30\,t - t^2$$

$$\boxed{x = 30\,t - t^2 \text{ m}}$$

$$v = v_o + at$$

$$\boxed{v = 30 - 2t \text{ m/s}}$$

Example 2 continue

2. Determine the maximum distance traveled by the car up the hill after stalling.

Solution

x reaches a maximum when v = 0 then,

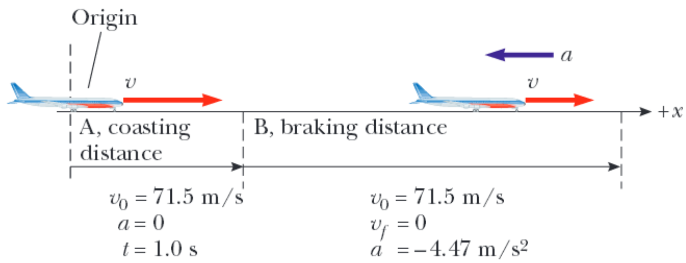
$$v = 30 - 2t = 0 \qquad \text{therefore} \qquad t = 15 \text{ s}$$

$$x_{max} = 30\,t - t^2$$

$$x = 30\,t - t^2 = 30\,(15) - (15)^2 = 225\text{m}$$

Example 3

A typical jetliner lands at a speed of 71.5m/s and decelerates at the rate of 4.47m/s^2. If the plane travels at a constant speed of 71.5m/s for 1.0 s after landing before applying the brakes, **what is the total displacement of the aircraft between touchdown on the runway and coming to rest?**



Solution

the displacement while the plane is coasting:

$$\Delta x_{\text{coasting}} = v_0 t + \tfrac{1}{2}at^2 = (71.5\text{ m/s})(1.00\text{ s}) + 0 = 71.5\text{ m}$$

the displacement while the plane is braking:

$$v^2 = v_0^2 + 2a\,\Delta x_{\text{braking}}$$

$$\Delta x_{\text{braking}} = \frac{v^2 - v_0^2}{2a} = \frac{0 - (71.5\text{ m/s})^2}{2.00(-4.47\text{ m/s}^2)} = 572\text{ m}$$

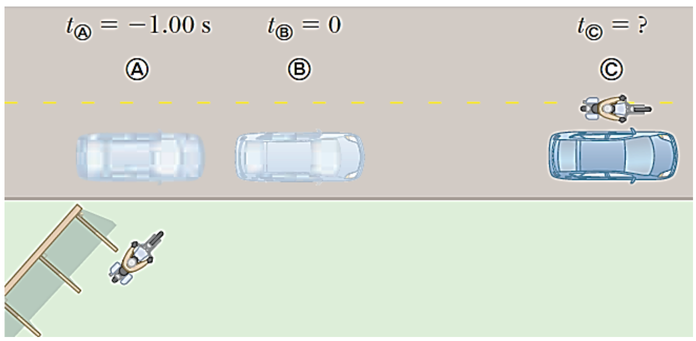
Sum the two results to find the total displacement:

$$\Delta x_{\text{coasting}} + \Delta x_{\text{braking}} = 72\text{ m} + 572\text{ m} = 644\text{ m}$$

Exercise: A jet lands at 80.0 *m/s*, the pilot applying the brakes 2.0 s after landing. Find the acceleration needed to stop the jet within 5.00 × 10^2 m. Ans. a = - 9.41 m/s^2.

Example 4

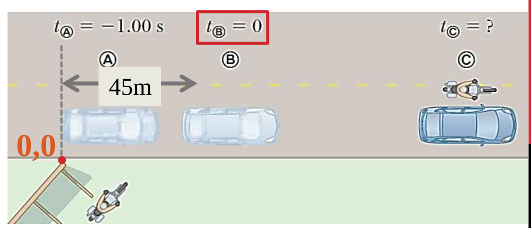
A car traveling at a constant speed of 45.0 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3.00 m/s^2. **(a) How long does it take her to overtake the car? (b) How fast is the trooper going at that time?**



Solution

(a) How long does it take her to overtake the car?

The position of each vehicle as a function of time.



$$x_{car} = x_Ⓑ + v_{x\,car}t$$

Fro the car (constant velocity)

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$
$$x_{trooper} = 0 + (0)t + \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}a_x t^2$$

Fro the trooper (constant acceleration)

$$x_{trooper} = x_{car}$$
$$\tfrac{1}{2}a_x t^2 = x_Ⓑ + v_{x\,car}t$$
$$\tfrac{1}{2}a_x t^2 - v_{x\,car}t - x_Ⓑ = 0$$

The positions of the car and trooper equal at position ©

$$t = \frac{v_{x\,car} \pm \sqrt{v_{x\,car}^2 + 2a_x x_Ⓑ}}{a_x}$$

Solve the quadratic equation for the time at which the trooper catches the car

Solution continue

$$t = \frac{v_{x\,car} \pm \sqrt{v_{x\,car}^2 + 2a_x x_{\text{Ⓑ}}}}{a_x}$$

$$t = \frac{v_{x\,car}}{a_x} \pm \sqrt{\frac{v_{x\,car}^2}{a_x^{\,2}} + \frac{2x_{\text{Ⓑ}}}{a_x}}$$

Evaluate the solution, choosing the positive root because that is the only choice consistent with a time $t > 0$

$$t = \frac{45.0\ \text{m/s}}{3.00\ \text{m/s}^2} + \sqrt{\frac{(45.0\ \text{m/s})^2}{(3.00\ \text{m/s}^2)^2} + \frac{2(45.0\ \text{m})}{3.00\ \text{m/s}^2}} = 31.0\ \text{s}$$

(b) How fast is the trooper going at that time?

$v_{trooper} = v_o + a_{trooper}t = 0 + (3.0 m/s^2)(31.0\ \text{s})$
$= 93$ m/s

لماذا لم نقم باختيار t=0 عندما تجاوزت السيارة الموتورسيكل؟ اذا قمنا بذلك فاننا لا نستطيع استخدام معادلات الحركة عند ثبات التسارع للموتورسيكل لان تسارعه كان يساوي صفر خلال الثانية الأولى وبعد ذلك اصبح التسارع $3\ m/s^2$. وبالتالي باعتبار الزمن t=0 عندما بدأ الموتورسيكل الحركة فاننا نستطيع استخدام معادلات الحركة عند ثبات العجلة.

Problems to be solved by yourself

1. A car traveling initially at a speed of 60m/s is accelerated uniformaly to a speed 85m/s in 12s. How far does the car travel during the 12s interval?
2. A body moving with uniform acceleration has a velocity of 12cm/s when its coordinate is 3cm. If its *x* coordinate 2s later is -5cm, what is the magnitude of its acceleration?
3. The initial speed of a body is 5.2m/s. What is its speed after 2.5s if it (a) accelerates uniformaly at $3m/s^2$ and (b) accelerates uniformaly at $-3m/s^2$.

المحاضرة القادمة

Free Fall Acceleration