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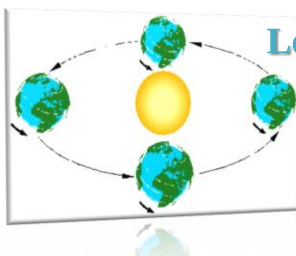
General Physics I

Mechanics: Principles & Applications

Unit6: The Law of Universal Gravitation



Lecture (20) Gravitational potential energy

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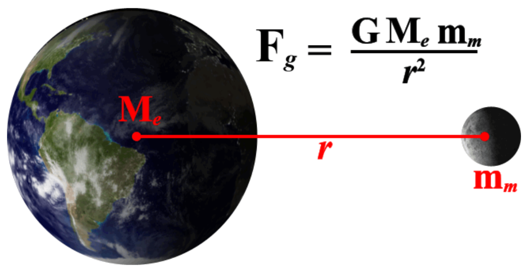
Unit 6: The Law of Universal Gravitation

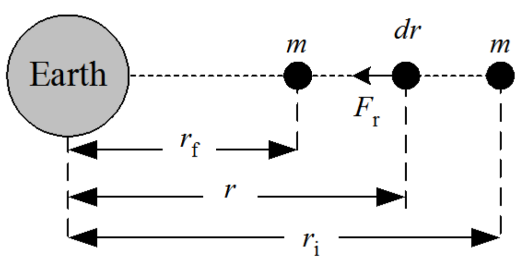
Gravitational Potential Energy

درسنا في محاضرة سابقة أن طاقة الوضع لجسم على سطح الأرض أو على ارتفاع *h* من سطح الأرض تساوي *mgh* وهذا عندما تكون *h* على مسافات قريبة من سطح الأرض أو عندما تكون *h* أصغر بكثير من نصف قطر الأرض.

سندرس الآن طاقة الوضع في مجال الجاذبية الأرضية عندما يتغير موضع الجسم من مكان إلى آخر بالنسبة لمركز الأرض كما في الشكل التالي.



To move the particle of mass m from r_i to r_f in the gravitational field g a negative work W is done by an external agent since the external force F_{ex} is in opposite direction of the displacement. Therefore the change in gravitational potential energy associated with a given displacement dr is defined as the negative work done by the gravitational force during the displacement,

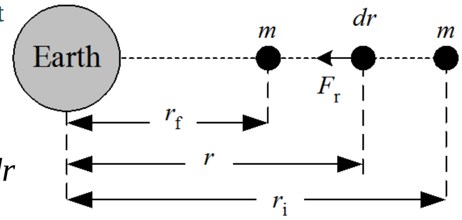


$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r) dr$$

When the particle move from r_i to r_f, it will be subjected to gravitational force given by

$$\vec{F} = -\frac{GM_e m}{r^2}$$

where the negative sign indicates that the force is attractive.



Substitute in equation

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr$$

$$U_f - U_i = GM_e m \int_{r_i}^{r_f} \frac{dr}{r^2} = GM_e m \left[-\frac{1}{r} \right]_{r_i}^{r_f}$$

$$U_f - U_i = -GM_e m \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

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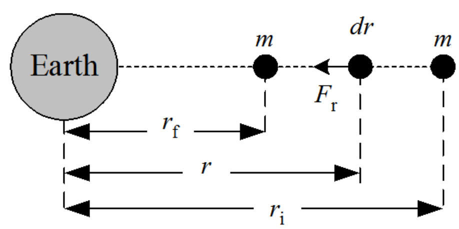


Take U_i=0 at r_i=∞ we obtain the potential energy as a function of r from the centre of the earth

$$U(r) = -\frac{GM_e m}{r}$$

The potential energy between any two particles m_1 and m_2 is given by

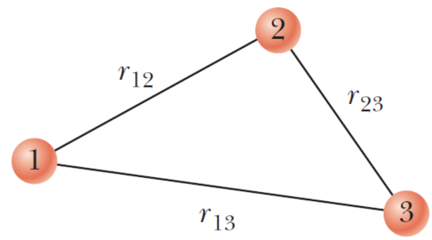
$$U = -G\frac{m_1 m_2}{r}$$

- نستنتج من المعادلة الأخيرة أن طاقة الوضع المتبادلة بين جسمين تتناسب عكسيا مع المسافة الفاصلة بينهما في حين أن قوة الجاذبية تتناسب عكسياً مع مربع المسافة بينهما.
- تكون طاقة الوضع بين جسمين سالبة لأن القوة المتبادلة بينهما دائماً قوى تجاذبية.

Potential Energy for more than Two Particles

For more than two particles the potential energy can be evaluated by the algebraic sum of the potential energy between any two particles.

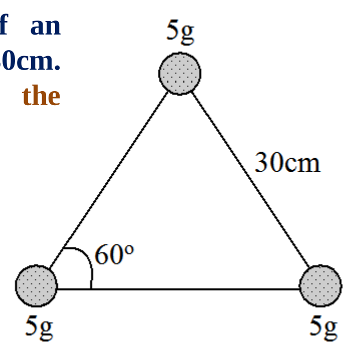
$$U_{total} = U_{12} + U_{13} + U_{23}$$



$$U_{total} = -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}}\right)$$

Example 1

A system consists of three particles, each of mass 5g, located at the corner of an equilateral triangle with sides of 30cm. Calculate the potential energy of the system.



Solution

$$U_{total} = U_{12} + U_{13} + U_{23}$$

$$U_{total} = -G\left(\frac{m^2}{r} + \frac{m^2}{r} + \frac{m^2}{r}\right) = -\frac{3GM^2}{r}$$

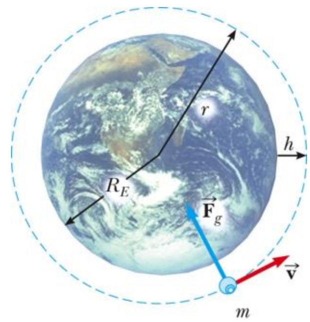
$$U_{total} = -\frac{3\times 6.67\times 10^{-11}\times (0.005)^2}{0.3} = -1.67\times 10^{-14}\,J$$

Total Energy for Circular Orbital Motion

When a body of mass *m* moving with speed *v* in circular orbit around another body of mass *M* where *M* >> *m* as the earth around the sun or satellite around the earth, the body of mass *M* is at rest with respect to the frame of reference. *The total energy of the two body system is the sum of the kinetic energy and the potential energy.*

$$E = K + U$$

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$



As the mass *m* moves from initial point *i* to a final point *f*, the total energy remains constant, therefore the total energy equation become,

$$E = \frac{1}{2}mv_i^2 - \frac{GMm}{r_i} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f}$$

From Newton's second law F = ma where *a* is the is the radial acceleration therefore,

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Multiply both sides by *r*/2

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$E = \frac{1}{2}mv_i^2 - \frac{GMm}{r_i} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f}$$

$$E = \frac{GMm}{2r} - \frac{GMm}{r}$$

The total energy for circular orbit ⟹ $E = -G\frac{Mm}{2r}$

Note that the total energy is negative in a circular orbit. And the kinetic energy is positive and equal to one half the magnitude of the potential energy. The total energy called the binding energy for the system.

Example 2

A space transportation vehicle releases a 470-kg communications satellite while in an orbit 280 km above the surface of the Earth. A rocket engine on the satellite boosts it into a geosynchronous orbit (المدار الأرضي الجغرافي المتزامن) 36,000 km above the surface of the Earth. How much energy does the engine have to provide?

Solution

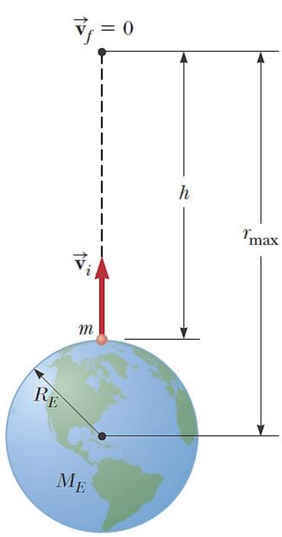
$$r_i = R_E + 280 \text{ km} = 6.65 \times 10^6 \text{ m}$$

$$\Delta E = E_f - E_i = -\frac{GM_E m}{2r_f} - \left(-\frac{GM_E m}{2r_i}\right) = -\frac{GM_E m}{2}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$\Delta E = -\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})(470 \text{ kg})}{2} \times \left(\frac{1}{4.22 \times 10^7 \text{ m}} - \frac{1}{6.65 \times 10^6 \text{ m}}\right)$$

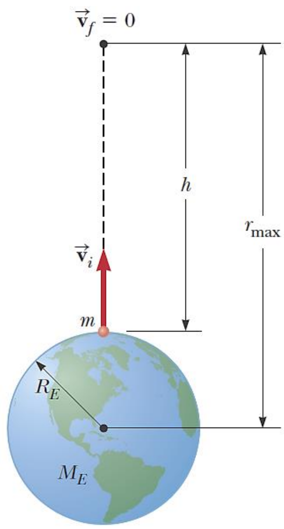
$$= 1.19 \times 10^{10} \text{ J}$$

Escape Velocity



باستخدام مفهوم الطاقة الكلية سنقوم بحساب سرعة الإفلات *escape velocity* من الجاذبية الأرضية. وسرعة الإفلات هي أقل سرعة ابتدائية لجسم يقذف رأسياً ليتمكن الجسم من الإفلات من مجال الجاذبية الأرضية.

Suppose an object of mass *m* is projected vertically upward from the earth with initial speed $v_i = v$ and $r_i = Re$. When the object is at maximum altitude, $v_f = 0$ and $r_f = r_{max}$.



In this case the total energy of the system (Earth & object) is conserved, we can use the equation

$$\frac{1}{2} m v_i^2 - \frac{G M_e m}{R_e} = -\frac{G M_e m}{r_{max}}$$

solving for v_i^2 we get,

$$v_i^2 = 2 G M_e \left(\frac{1}{R_e} - \frac{1}{r_{max}} \right)$$

من هذه المعادلة إذا علمنا قيمة السرعة الابتدائية لانطلاق الجسم v_i يمكن حساب أقصى ارتفاع يمكن أن يصل إليه الجسم h حيث أن $h = r_{max} - Re$.

لحساب سرعة الإفلات للجسم من مجال الجاذبية الأرضية مثل ما هو الحال عند إطلاق صاروخ فضائي أو مكوك من سطح الأرض إلى الفضاء الخارجي فإن سرعة الانطلاق الابتدائية التي يجب أن ينطلق بها المكوك يجب أن لا تقل عن سرعة الإفلات وإلا فإن المكوك سوف لن يصل إلى هدفه نتيجة لتأثير قوة الجاذبية. ولإيجاد سرعة الإفلات المطلوبة فإن ......

$$v_i^2 = 2 G M_e \left(\frac{1}{R_e} - \frac{1}{r_{max}} \right)$$

For the escape velocity the object will reach a final speed of $v_f = 0$ when $r_{max} = \infty$, therefore we substitute for $v_i = v_{esc}$ and we get

$$v_{esc} = \sqrt{\frac{2 G M_e}{R_e}}$$

Note that the escape velocity does not depends on the mass of the object projected from the earth.

This equation can be used to evaluate the escape velocity from any planet in the universe if the mass and the radius of the planet are known.

$$v_{esc} = \sqrt{\frac{2GM_e}{R_e}}$$

Escape velocities for the planets	
Planet	v_{esc} (km/s)
Mercury	4.3
Venus	10.3
Earth	11.2
Moon	2.3
Mars	5.0
Jupiter	60
Saturn	36
Uranus	22
Neptune	24
Pluto	1.1
Sun	618

Example 2

Calculate the escape speed from the Earth for a 5000-kg spacecraft and **determine** the kinetic energy it must have at the Earth's surface to move infinitely far away from the Earth.

Solution

$$v_{\text{esc}} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})}{6.37 \times 10^6\ \text{m}}}$$

$$= 1.12 \times 10^4\ \text{m/s}$$

$$K = \tfrac{1}{2}mv_{\text{esc}}^2 = \tfrac{1}{2}(5.00 \times 10^3\ \text{kg})(1.12 \times 10^4\ \text{m/s})^2$$

$$= 3.13 \times 10^{11}\ \text{J}$$

Example 3

(a) Calculate the minimum energy required to send a 3000kg spacecraft from the earth to a distance point in space where earth's gravity is negligible. (b) If the journey is to take three weeks, what average power would the engine have to supply?

Solution

(a)

$$v_{esc} = \sqrt{\frac{2GM_e}{R_e}} = 1.12\times10^4\,\text{m/s}$$

$$K = \frac{1}{2}mv_{esc}^2 = \frac{1}{2}\times3000\times(1.12\times10^4)^2$$

$$= 1.88\times10^{11}\,J$$

(b)

$$P_{av} = \frac{K}{\Delta t} = \frac{1.88\times10^{11}}{21\,\text{days}\times8.64\times10^4\,s/\text{day}} = 103kW$$

Example 4

A spaceship is fired from the Earth's surface with an initial speed of 2×10^4 m/s. What will its speed when it is very far from the Earth? (Neglect friction.)

Solution

Energy is conserved between the surface and the distant point

$$(K+U_g)_i = (K+U_g)_f$$

$$\frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = \frac{1}{2}mv_f^2 - \frac{GM_E m}{\infty}$$

$$v_f^2 = v_i^2 - \frac{2GM_E}{R_E} \quad\Longrightarrow\quad v_f^2 = v_i^2 - v_{esc}^2$$

$$v_f^2 = (2\times10^4)^2 - \frac{2(6.67\times10^{-11})^2(5.98\times10^{24})}{6.37\times10^6}$$

$$v_f = 1.66\times10^4 \quad m/s$$

Problems to be solved by yourself

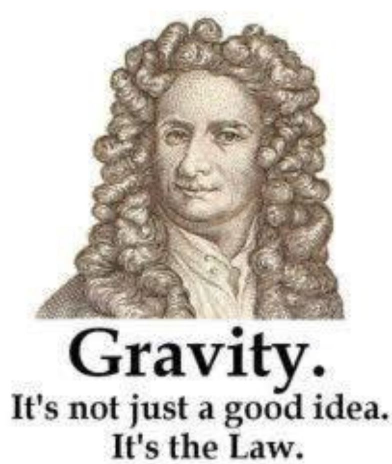
1. A satellite of the earth has a mass of 100 kg and is altitude of 2×10^6 m. (a) What is the potential energy of the satellite-earth system? (b) What is the magnitude of the force on the satellite?

2. A system consists of three particles, each of mass 5g, located at the corners of an equilateral triangle sides of 30 cm. (a) Calculate the potential energy of the system. (b) If the particles are released simultaneously, where will they collide?

3. How much energy is required to move a 1000-kg form the earth's surface to an altitude equal to twice the earth's radius?

4. Calculate the escape velocity from the moon, where $M_m=7.36\times10^{22}$kg, $R_m=1.74\times10^6$m

5. A spaceship is fired from the earth's surface with an initial speed of 2.0×10^4m/s. What will its speed when it is very far from the earth?



المحاضرة القادمة

Problems on Gravitational Potential Energy