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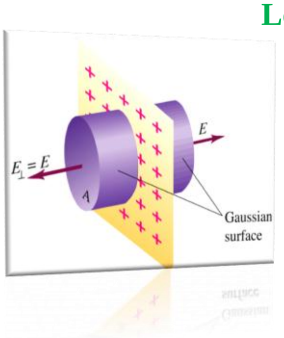
General Physics II

Electrostatic: Principles & Applications

Lecture (7): Gauss's Law Applications



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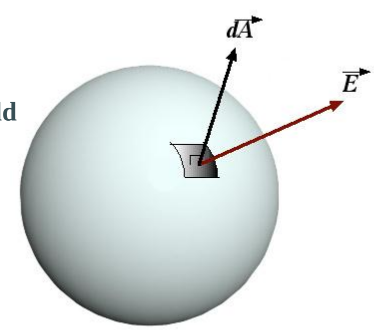
Electric Flux



4.1 The Electric Flux due to an Electric Field
4.2 The Electric Flux due to a point charge
4.3 Gaussian surface
4.4 Gauss's Law
4.5 Gauss's law and Coulomb's law
4.6 Conductors in electrostatic equilibrium
4.7 Applications of Gauss's law
4.8 Solution of some selected problems
4.9 Problems

في هذه المحاضرة سوف نقوم بدراسة العديد من التطبيقات لقانون جاوس مثل تأثير الشحنة الكهربية على الموصل المعزول وسوف نتعلم كيف نحسب المجال الكهربي لتوزيع متصل من الشحنة على شكل خطي أو سطحي أو حجمي.

Conductors in Electrostatic Equilibrium

A good electrical conductor, such as copper, contains charges (electrons) that are free to move within the material. When there is no net motion of charges within the conductor, the conductor is in electrostatic equilibrium.

Conductor in electrostatic equilibrium has the following properties:

Any excess charge on an isolated conductor must reside entirely on its surface. *(Explain why?)* ***The answer is when an excess charge is placed on a conductor,***

It will set-up electric field inside the conductor.

These *fields act on the charge carriers of the conductor (electrons) and cause them to move* ***i.e. current flow inside the conductor.***

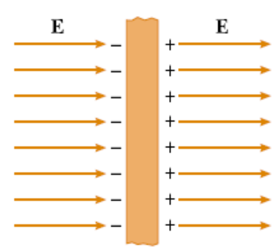
These currents redistribute the excess charge on the surface in such away that the internal electric fields reduced to become zero and the currents stop, and the electrostatic conditions restore.

The electric field is zero everywhere inside the conductor. ***(Explain why?) Same reason as above.***

Conducting Slab in an External Electric Field

In the figure it shows a conducting slab in an external electric field *E*.

The charges induced on the surface of the slab produce an electric field, which opposes the external field, giving a resultant field of zero in the conductor.



في الشكل المقابل تم وضع شريحة معدنية موصلة في مجال كهربي خارجي ماذا يحدث؟

مادة الشريحة موصلة وهذا يعني أن الشحنات حرة الحركة فتتحرك الشحنات إلى السطح الخارجي كما في الشكل لينتج عنها مجالاً كهربيا يعاكس المجال الكهربي الخارجي وهذا يعني أن المجال الكهربي داخل مادة الموصل تساوي صفر.

دائما تذكر إن المجال الكهربي داخل مادة الموصل تساوي صفر والشحنة الكهربية الإضافية تستقر على السطح الخارجي للموصل.

Applications of Gauss's Law

Gauss's law can be used to calculate the electric field if the symmetry of the charge distribution is high. Here we concentrate in three different ways of charge distribution

يستخدم قانون جاوس لحساب المجال الكهربي الناتج عن توزيع متصل للشحنة حيث يصعب إيجاد المجال الكهربي باستخدام قانون كولوم. ومن أمثلة التوزيع المتصل للشحنة سلك مشحون أو سطح لانهائي مشحون أو كرة مشحونة، وفي هذه الحالات نفترض إن توزيع الشحنة هو توزيع متجانس ونعبر عنه بكثافة الشحنة.

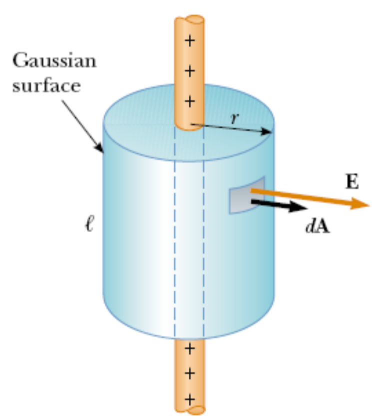
تكون كثافة الشحنة charge density على النحو التالي:

3	2	1	
Volume	Surface	Linear	Charge distribution
ρ	σ	λ	Charge density
C/m^3	C/m^2	C/m	Unit

A Linear Charge Distribution

In the figure calculate the electric field at a distance *r* from a uniform positive line charge of infinite length whose charge per unit length is λ=constant.

The electric field E is perpendicular to the line of charge and directed outward. Therefore for symmetry we select a cylindrical gaussian surface of radius r and length L.



The electric field is constant in magnitude and perpendicular to the surface.

The flux through the end of the gaussian cylinder is zero since E is parallel to the surface.

The total charge inside the gaussian surface is λL.

A Linear Charge Distribution, *Continue*

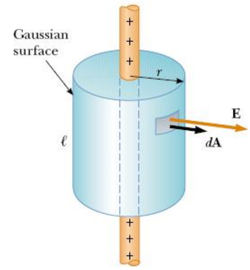
Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$ نطبق قانون جاوس على سطح اسطواني يحيط بالسلك

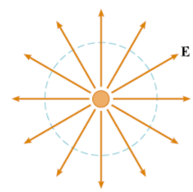
$$E\oint dA = \frac{\lambda L}{\varepsilon_o}$$ نعوض عن الشحنة بكثافة الشحنة

$$E2\pi rL = \frac{\lambda L}{\varepsilon_o}$$ نجري عملية التكامل

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_o r}$$



(a)



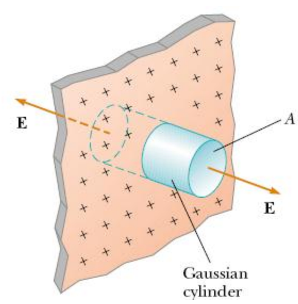
(b)

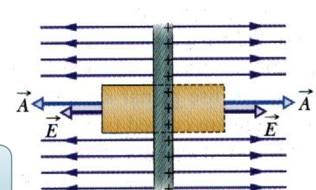
نلاحظ هنا أنه باستخدام قانون جاوس سنحصل على نفس النتيجة التي توصلنا لها بتطبيق قانون كولوم وبطريقة أسهل.

A Surface Charge Distribution

In the figure calculate the electric field due to non-conducting, infinite plane with uniform charge per unit area σ.

The electric field E is constant in magnitude and perpendicular to the plane charge and directed outward for both surfaces of the plane. Therefore for symmetry we select a cylindrical gaussian surface with its axis is perpendicular to the plane, each end of the gaussian surface has area A and are equidistance from the plane.
The flux through the end of the gaussian cylinder is EA since E is perpendicular to the surface.
The total electric flux from both ends of the gaussian surface will be $2EA$.





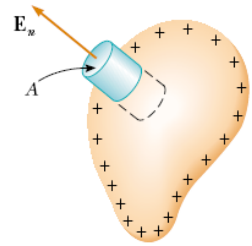
Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \rightarrow 2EA = \frac{\sigma A}{\varepsilon_o} \rightarrow \therefore E = \frac{\sigma}{2\varepsilon_o}$$

An Insulated Conductor

ذكرنا سابقا أن الشحنة توزع على سطح الموصل فقط، وبالتالي فإن قيمة المجال داخل مادة الموصل تساوى صفراً، وقيمة المجال خارج الموصل تساوى

$$E = \frac{\sigma}{\varepsilon_o}$$



في الشكل الموضح أعلاه نلاحظ أن الوجه الأمامي لسطح جاوس له فيض حيث أن الشحنة تستقر على السطح الخارجي، بينما يكون الفيض مساوياً للصفر للسطح الخلفي الذي يخترق الموصل وذلك لأن الشحنة داخل الموصل تساوي صفراً.

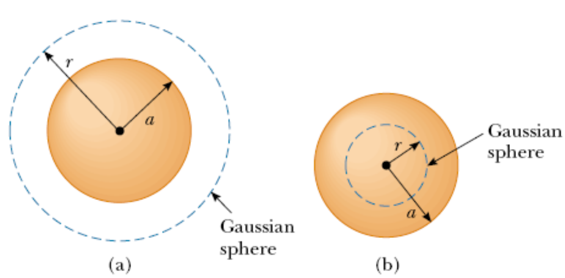
لاحظ هنا أن المجال في حالة الموصل يساوى ضعف قيمة المجال في حالة السطح اللانهائي المشحون، وذلك لأن خطوط المجال تخرج من السطحين في حالة السطح غير الموصل، بينما كل خطوط المجال تخرج من السطح الخارجي في حالة الموصل.

A Volume Charge Distribution

In the figure shows an insulating sphere of radius *a* has a uniform charge density ρ and a total charge *Q*.

1) Find the electric field at point outside the sphere (*r*>*a*)

2) Find the electric field at point inside the sphere (*r*<*a*)



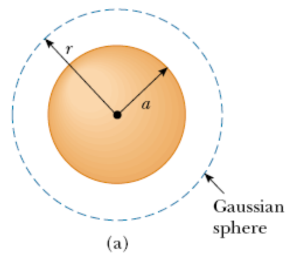
1) Find The Electric Field at Point Outside the Sphere (*R*>*a*)

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where *r*>*a*. The electric field *E* is perpendicular to the gaussian surface as shown in figure. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint A = E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon_o r^2} \quad \text{(for r>a)}$$



(a)

Note that the result is identical to appoint charge.

2) Find The Electric Field at Point Inside the Sphere (*R*<*a*)

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where *r*<*a*. The electric field *E* is perpendicular to the gaussian surface as shown in figure 4.17. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

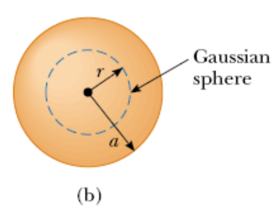
It is important at this point to see that the charge inside the gaussian surface of volume $V`$ is less than the total charge Q.

To calculate the charge q_{in}, we use $q_{in}=\rho V`$, where $V`=4/3\pi r^3$. Therefore,

$$q_{in} = \rho V` = \rho(4/3\pi r^3)$$

$$E\oint A = E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_o r^2} = \frac{\rho}{3\varepsilon_o} r$$

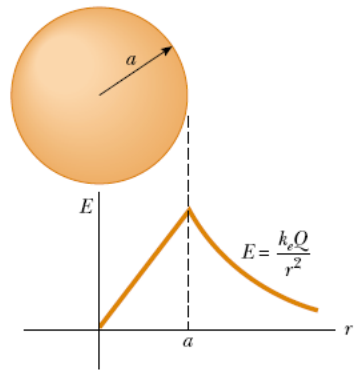


(b)

$$E=\frac{q_{in}}{4\pi\varepsilon_o r^2}=\frac{\rho\frac{4}{3}\pi r^3}{4\pi\varepsilon_o r^2}=\frac{\rho}{3\varepsilon_o}r$$

since $\rho=\frac{Q}{\frac{4}{3}\pi a^3}$

$$\therefore E=\frac{Qr}{4\pi\varepsilon_o a^3}$$ (for $r<a$)



Note that the electric field when $r<a$ is proportional to r, and when $r>a$ the electric field is proportional to $1/r^2$.

Steps Which Should be Followed in Solving Problems

1. **The gaussian surface should be chosen to have the same symmetry as the charge distribution.**
2. **The dimensions of the surface must be such that the surface includes the point where the electric field is to be calculated.**
3. **From the symmetry of the charge distribution, determine the direction of the electric field and the surface area vector *dA*, over the region of the gaussian surface.**
4. **Write *E.dA* as *E dA cos*θ and divide the surface into separate regions if necessary.**
5. **The total charge enclosed by the gaussian surface is *dq* = ∫*dq*, which is represented in terms of the charge density (*dq* = λ*dx* for line of charge, *dq* = σ*dA* for a surface of charge, *dq* = ρ*dv* for a volume of charge).**