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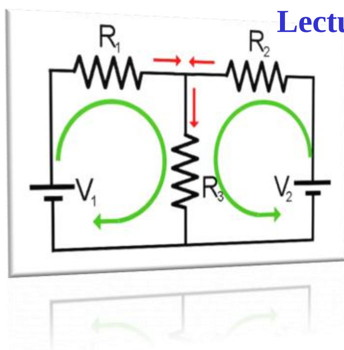
General Physics II

Electrostatic: Principles & Applications



Lecture (20): Direct Current Circuits
Kirchhoff's Rules

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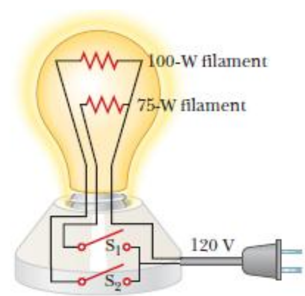
Direct Current Circuits

- Electromotive Force
- Finding the current in a simple circuit
- Kirchhoff's Rules
- Single-Loop Circuit
- Multi-Loop Circuit
- Problems



سنتعامل في هذا الفصل مع الدوائر الكهربية التي تحتوي على بطارية ومقاومة. سنقوم بتحليل هذه الدوائر الكهربية معتمدين على قاعدة كيرشوف Kirchhoff's rule لحساب التيار الكهربي المار في كل عنصر من عناصر الدائرة الكهربية. وبداية سنتعرف على مفهوم القوة الدافعة الكهربية Electromotive force.

Kirchhoff's Rules

A practical electrical circuit is usually complicated system of many electrical elements. Kirchhoff extended Ohm's law to such systems, and gave two rules, which together enabled the current in any part of the circuit to be calculated.

Statements of Kirchhoff's Rules

(1) The algebraic sum of the currents entering any junction must equal the sum of the currents leaving that junction.

$$\sum_i I_i = 0 \quad \text{at the junction}$$

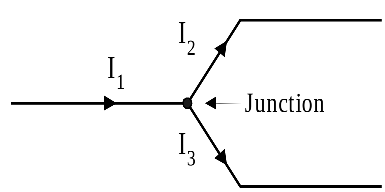
(2) The algebraic sum of the changes in potential difference across all of the elements around any closed circuit loop must be zero.

$$\sum_i \Delta V_i = 0 \quad \text{for the loop circuit}$$

Note that the first Kirchhoff's rule is for the current and the second for the potential difference.

Kirchhoff's Rules

Applying the first rule on the junction shown below

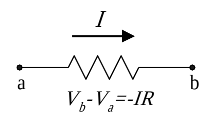


$$I_1 = I_2 + I_3$$

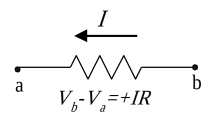
Kirchhoff's Rules

Applying the second rule on the following cases

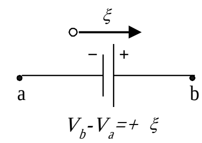
(1) If a resistor is traversed in the direction of the current, the change in potential difference across the resistor is *-IR*.



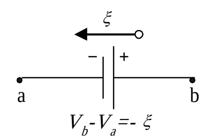
(2) If a resistor is traversed in the direction opposite the current, the change in potential difference across the resistor is *+IR*.



(3) If a source of *emf* is traversed in the direction of the *emf* (from - to + on the terminal), the change in potential difference is + .



(4) If a source of *emf* is traversed in the direction opposite the *emf* (from + to - on the terminal), the change in potential difference is - .



Hints for solution of problems using kirchhoff's rules

لاستخدام قاعدة كيرشوف يجب اتباع الخطوات التالية:

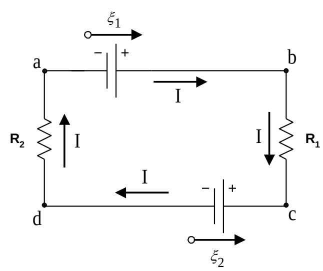
1. حدد اتجاه القوة الدافعة الكهربية *emf* لكل بطارية في الدائرة الكهربية بسهم متجه من القطب السالب إلى القطب الموجب للبطارية.
2. حدد اتجاه التيار الكهربائي لكل عنصر من عناصر الدائرة الكهربية مثل المقاومة بافتراض اتجاه محدد للتيار حتى تتمكن من تطبيق قاعدتي كيرشوف. فإذا كان الحل النهائي يظهر إشارة موجبة للتيار يكون الاتجاه المفترض صحيحاً، أما إذا ظهرت إشارة التيار سالبة فإن قيمة التيار صحيحة، ولكن اتجاه التيار في الاتجاه المعاكس للاتجاه المفترض.
3. نطبق القاعدة الأولى لكيرشوف عند العقدة الموجودة في الدائرة الكهربية بحيث تكون إشارة التيارات الداخلة على العقدة موجبة والخارجة من العقدة سالبة.
4. نطبق القاعدة الثانية لكيرشوف على مسار مغلق محدد لكل فرع من أفرع الدائرة الكهربية ونراعي التغير في فرق الجهد على كل عنصر من عناصر الدائرة الكهربية إذا كان سالباً أو موجباً.
5. نحل المعادلات الرياضية التي نتجت من تطبيق الخطوتين (3) و(4) حلاً جبرياً.

Single-loop Circuit

In a single-loop circuit there is no junctions and the current is the same in all elements of the circuit, therefore we use only the second Kirchhoff rule.

Example (1)

Two battery are connected in opposite in a circuit contains two resistors as shown in figure 8.6 the *emf* and the resistance are ξ_1=6v, ξ_2=12v, R_1= 8Ω, and R_2=10 Ω. (a) Find the current in the circuit. (b) What is the power dissipated in each resistor?



Solution

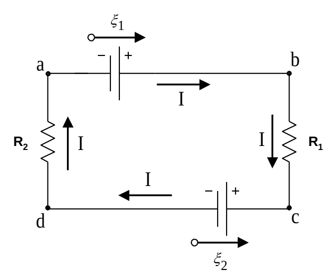
If we assume the current in the circuit is in the clockwise direction (*abcda*). Applying the second Kirchhoff's rule along arbitrary loop (*abcda*) we get

$$\sum_i \Delta V_i = 0$$

$$+\xi_1 - IR_1 - \xi_2 + IR_2 = 0$$

Solving for the current we get

$$I = \frac{\zeta_1 - \xi_2}{R_1 + R_2} = \frac{6-12}{8+10} = -\frac{1}{3}A$$



The -ve sign of the current indicates that the correct direction of the current is opposite the assumed direction *i.e.* along the loop (*adba*)

The power dissipated in R_1 and R_2 is

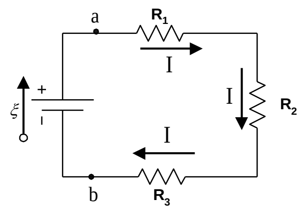
$$P_1 = I^2 R_1 = 8/9\text{W}$$

$$P_2 = I^2 R_2 = 10/9\text{W}$$

In this example the battery ξ_2 is being charged by the battery ξ_1.

Example (2)

Three resistors are connected in series with battery as shown in the figure, apply second Kirchhoff's rule to (a) Find the equivalent resistance and (b) find the potential difference between the points *a* and *b*.



Solution

Applying second Kirchhoff's rule in clockwise direction we get

$$-IR_1 - IR_2 - IR_3 + \xi = 0$$

$$I = \frac{\zeta}{R_1 + R_2 + R_3}$$

$$\because I = \frac{\zeta}{R}$$

therefore,

$$R = R_1 + R_2 + R_3$$

To find the potential difference between points a and b V_{ab} ($=V_a - V_b$) we use the second Kirchhoff's rule along a direction starting from point (b) and finish at point (a) through the resistors. We get

$$V_b + IR = V_a$$

Where *R* is the equivalent resistance for R_1, R_2 and R_3

$$V_{ab} = V_a - V_b = +IR$$

The +ve sign for the answer means that $V_a > V_b$

Substitute for the current *I* using the equation $I = \frac{\zeta}{R}$

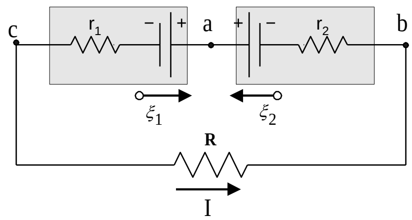
we get

$$V_{ab} = \xi$$

This means that the potential difference between points *a* and *b* is equal to the *emf* in the circuit (when the internal resistance of the battery is neglected).

Example (3)

In the circuit shown in the figure let ξ_1 and ξ_2 be 2v and 4v, respectively; r_1, r_2 and R be 1Ω, 2Ω, and 5Ω, respectively. (a) What is the current in the circuit? (b) What is the potential difference V_a-V_b and V_a-V_c?



Solution

$$-\xi_2 + Ir_2 + IR + Ir_1 + \xi_1 = 0$$

Solving the equation for the current we get

$$I = \frac{\zeta_2 - \xi_1}{R + r_1 + r_2} = \frac{4-2}{5+1+2} = +0.25A$$

The potential difference V_a-V_b we apply second Kirchhoff's rule starting at point *b* to finishing at point *a*.

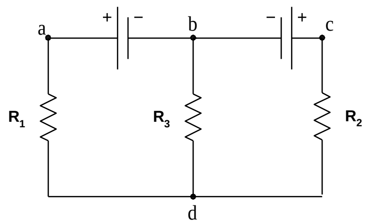
$$V_a - V_b = -Ir_2 + \xi_2 = (-0.25\times2)+4 = +3.5\text{v}$$

The potential difference V_a-V_c we apply second Kirchhoff's rule starting at point *c* to finishing at point *a*.

$$V_a - V_c = +\xi_1 + Ir_1 = +2+(-0.25\times1) = +2.25\text{v}$$

Multi-loop Circuit

Some circuits involving more than one current loop, such as the one shown in the figure. Here we have a circuit with three loops: a left inside loop, a right inside loop, and an outside loop.

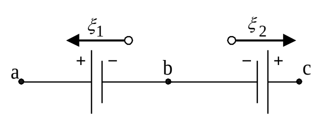


In the circuit shown above there are two junctions *b* and *d* and three branches connecting these junctions. These branches are bad, *bcd*, and *bd*. The problem here is to find the currents in each branch.

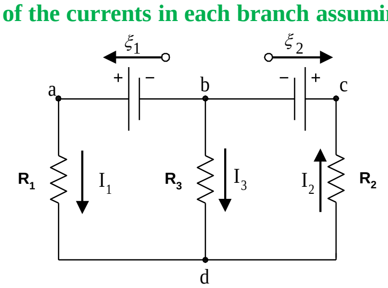
A general method for solving multi-loop circuit problem is to apply Kirchhoff's rules.

You should always follow these steps:

(1) Assign the direction for the *emf* from the -ve to the +ve terminal of the battery.



(2) Assign the direction of the currents in each branch assuming arbitrary direction.



After solving the equations the +ve sign of the current means that the assumed direction is correct, and the -ve sign for the current means that the opposite direction is the correct one.

(3) Chose one junction to apply the first Kirchhoff's rule.

$$\sum_i I_i = 0$$

At junction *d* current I_1 and I_3 is approaching the junction and I_2 leaving the junction therefore we get this equation

$$I_1 + I_3 - I_2 = 0 \qquad (1)$$

(4) For the three branches circuit assume there are two single-loop circuits and apply the second Kirchhoff's rule on each loop.

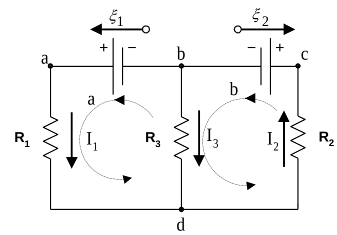
For loop a on the left side starting at point b we get

$$+\xi_1 - I_1R_1 + I_3R_3 = 0 \qquad (2)$$

For loop b on the left side starting at point b we get

$$- I_3R_3 - I_2R_2 - \xi_2 = 0 \qquad (3)$$

Equations (1), (2), and (3) can be solved to find the unknowns currents I_1, I_2, and I_3.



Example (4)

In the circuit shown in the figure, find the unknown current *I*, resistance *R*, and *emf* ξ.

Solution

At junction a we get this equation

$$I + 1 - 6 = 0$$

Therefore the current

$$I = 5\text{A}$$

To determine R we apply the second Kirchhoff's rule on the loop (a), we get

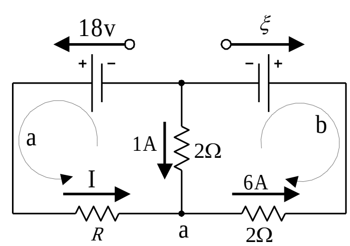
$$18 - 5R + 1 \times 2 = 0$$

$$R = 4\Omega$$

To determine ξ we apply the second Kirchhoff's rule on the loop (a), we get

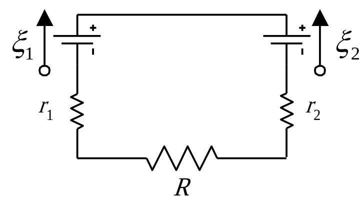
$$\xi + 6 \times 2 + 1 \times 2 = 0$$

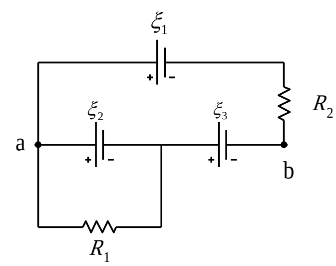
$$\xi = -14\text{v}$$



Problem to solve by your self

1. (*a)* In the figure what value must R have if the current in the circuit is to be 0.0010A? Take & ξ_1=2.0V, ξ_2=3.0V, and r_1=r_2=3.0Ω. (b) What is the rate of thermal energy transfer in R?

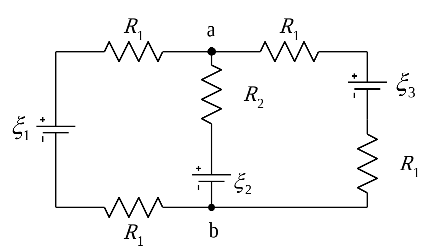


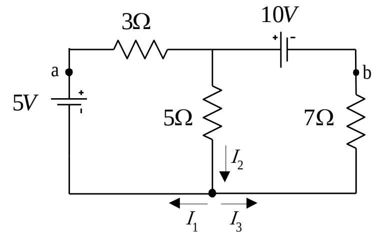


2. In the figure find the current in each resistor and the potential difference between a and b. Put ξ_1=6.0V, ξ_2=5.0V, $_3$=4.0V, R_1=100Ω and R_2 =50Ω.

Problem to solve by your self

3. (a) Find the three currents in the figure. (b) Find V_{ab}. Assume that R_1=1.0Ω, $R_{2=}$2.0Ω, ξ_1=2.0 V, and ξ_2=ξ_3=4.0V.





4. (a) Find the potential difference between points a and b in the circuit in the figure. (b) Find the currents I_1, I_2, and I_3 in the circuit