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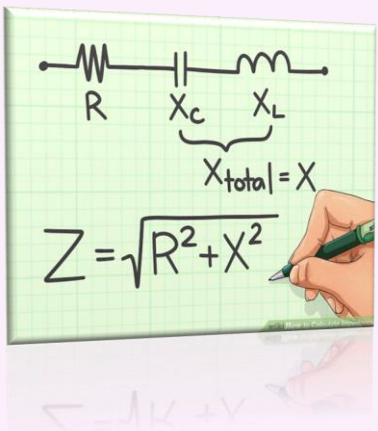
Electric Circuits



Unit: 9 | Lecture: 34

Sinusoids and Phasors
Impedance and Admittance

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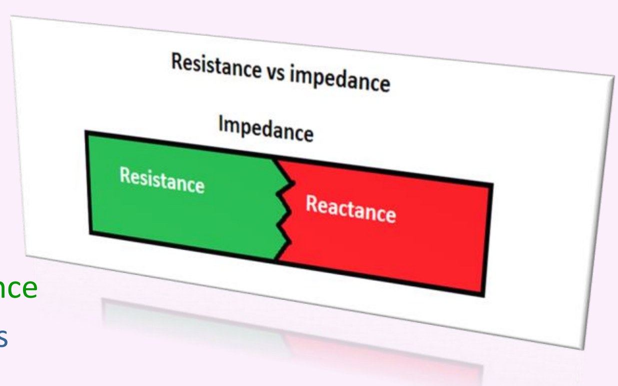
Sinusoids and Phasors



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9.5 Impedance (1)

The voltage-current relations for the three passive elements as

$\boldsymbol{V} = R\boldsymbol{I}$	$\boldsymbol{V} = j\omega L\boldsymbol{I}$	$\boldsymbol{V} = \dfrac{\boldsymbol{I}}{j\omega C}$
resistor	inductor	capacitor

These equations may be written in terms of the ratio of the phasor voltage to the phasor current as

$$\frac{\boldsymbol{V}}{\boldsymbol{I}} = R \qquad \frac{\boldsymbol{V}}{\boldsymbol{I}} = j\omega L \qquad \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{1}{j\omega C}$$

Ohm's law in phasor form for any type of element as

$$\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} \quad \text{or} \quad \boldsymbol{V} = \boldsymbol{Z}\boldsymbol{I}$$

where **Z** is a frequency-dependent quantity known as *impedance*, measured in ohms.

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9.5 Impedance (2)

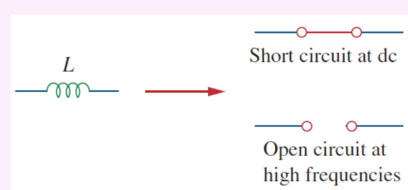
The impedance **Z** of a circuit is the ratio of the phasor voltage **V** to the phasor current **I**, measured in ohms (Ω).

Extreme case (1)

When $\omega = 0$ i.e. for dc sources

$$\boldsymbol{Z_L} = 0 \quad \text{and} \quad \boldsymbol{Z_C} \to \infty$$

confirming that the inductor acts like a short circuit, while the capacitor acts like an open circuit.

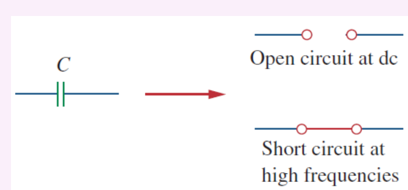


Extreme case (2)

When $\omega \to \infty$ i.e. for high frequencies

$$\boldsymbol{Z_L} \to \infty \quad \text{and} \quad \boldsymbol{Z_C} = 0$$

indicating that the inductor is an open circuit to high frequencies, while the capacitor is a short circuit.



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9.5 Impedance (3)

The impedance may be expressed in rectangular form as

$$\mathbf{Z} = R + jX$$

Where $R = \text{Re}\,\mathbf{Z}$ is the resistance and $X = \text{Im}\,\mathbf{Z}$ is the reactance

The reactance X may be positive or negative.

Thus, impedance

- $\mathbf{Z} = R + jX$ is said to be *inductive* or lagging since current lags voltage
- $\mathbf{Z} = R - jX$ is capacitive or leading because current leads voltage.

9.5 Impedance (4)

The impedance may also be expressed in polar form as

$$\mathbf{Z} = |\mathbf{Z}|\angle\theta$$

Therefore, the impedance is

$$\mathbf{Z} = R + jX \quad = |\mathbf{Z}|\angle\theta$$

where

$$|\mathbf{Z}| = \sqrt{R^2 + X^2} \qquad \theta = \tan^{-1}\frac{X}{R}$$

and

$$R = |\mathbf{Z}|\cos\theta \qquad X = |\mathbf{Z}|\sin\theta$$

9.5 Admittance (1)

The admittance **Y** is the reciprocal of impedance, measured in siemens (S).

$$\boldsymbol{Y} = \frac{\mathbf{1}}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}}$$

The admittance **Y** is the ratio of the phasor current through it to the phasor voltage across it

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

9.5 Admittance (2)

As a complex quantity the admittance **Y** may written as

$$\boldsymbol{Y} = G + jB$$

Where $G = \mathrm{Re}\,\boldsymbol{Y}$ is the conductance and $B = \mathrm{Im}\,\boldsymbol{Y}$ is the *susceptance*.

Admittance, conductance, and susceptance are all expressed in the unit of siemens (or mhos).

9.5 Admittance (3)

Relation between admittance and impedance

$\boldsymbol{Z} = R + jX$ impedance

$\boldsymbol{Y} = G + jB$ admittance

$$\boldsymbol{Y} = \frac{\boldsymbol{1}}{\boldsymbol{Z}}$$

$$G + jB = \frac{1}{R + jX}$$

$$G + jB = \frac{1}{R + jX} \times \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2}$$

Equating the real and imaginary parts gives

$$G = \frac{R}{R^2 + X^2} \qquad B = -\frac{X}{R^2 + X^2}$$

showing that $G \neq 1/R$ as it is in resistive circuits. Of course, if $X = 0$, then $G = 1/R$.

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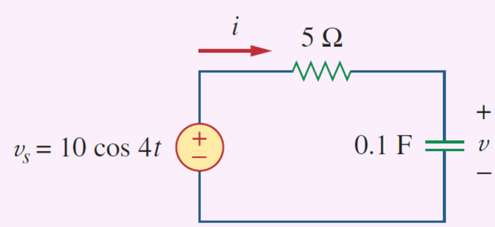
Example 9.9 (1)



Find $v(t)$ and $i(t)$ in the circuit

From the voltage source $v_s = 10 \cos 4t$

$$V_s = 10\angle 0$$

The impedance is

$$\boldsymbol{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

$$(5 - j2.5) \rightarrow r\angle\phi$$

$$r = \sqrt{5^2 + 2.5^2} = 5.6$$

$$\phi = \tan^{-1}\frac{-2.5}{5} = -26.57^o$$

Hence the current

$$\boldsymbol{I} = \frac{\boldsymbol{V_s}}{\boldsymbol{Z}} = \frac{10\angle 0}{5.6\angle - 26.5^o}$$

$$\boldsymbol{I} = 1.78\angle 26.57^o \text{ A}$$

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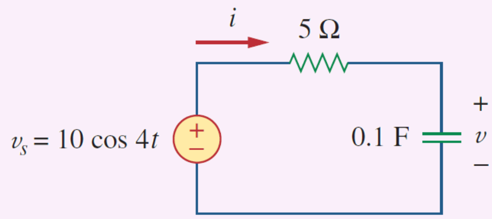
Example 9.9 (2)

The voltage across the capacitor is

$$\boldsymbol{V} = \boldsymbol{I}\boldsymbol{Z}_C = \frac{\boldsymbol{I}}{j\omega C} = \frac{1.78\angle 26.57^o}{j \times 4 \times 0.1}$$

$$= \frac{1.78\angle 26.57^o}{0.4\angle 90^o}$$

$$= 4.4\angle -63.42^o \text{ V}$$



Converting **I** and **V** to the time domain, we get

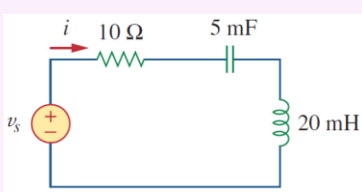
$$i(t) = 1.789\cos(4t + 26.57^o)\ \text{A}$$

$$v(t) = 4.47\cos(4t - 63.43^o)\ \text{V}$$

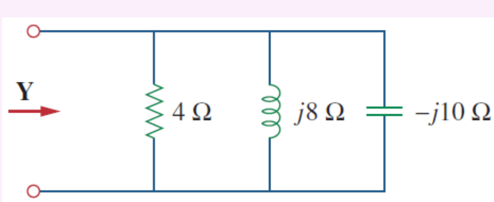
Notice that *i*(*t*) leads *v*(*t*) by 90 as expected.

Problems to Solve by yourself

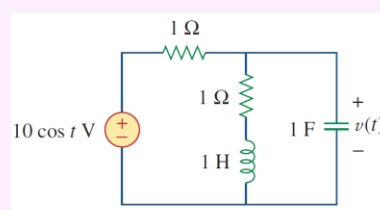
(1) Find current i in the circuit when $v_s(t) = 50\cos 200t$ V.



(2) Determine the admittance **Y** for the circuit



(3) Find *v*(*t*) in the *RLC* circuit



(4) If $V_o = 8\angle 30^o$ V in the circuit, find **I**s.