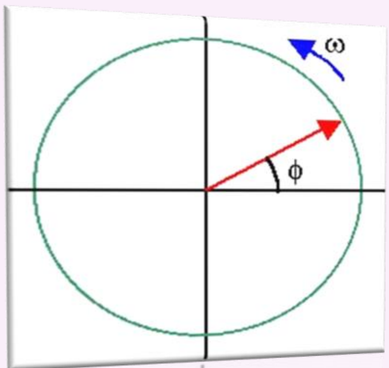
Physics Academy

www.physicsacademy.org

Electric Circuits



Unit: 9 | Lecture: 32

Sinusoids and Phasors

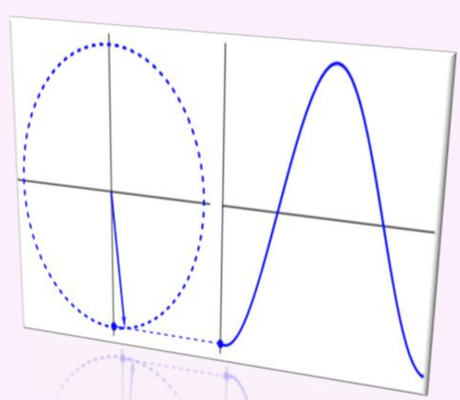
Phasors

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

1

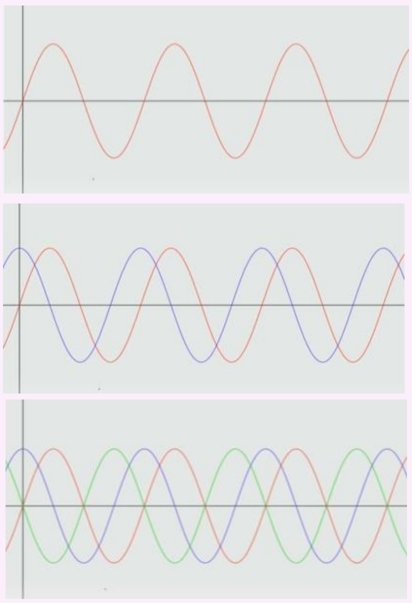
1

Sinusoids and Phasors

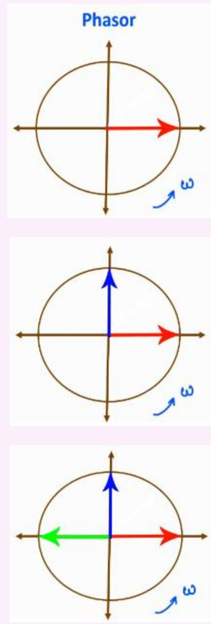


2

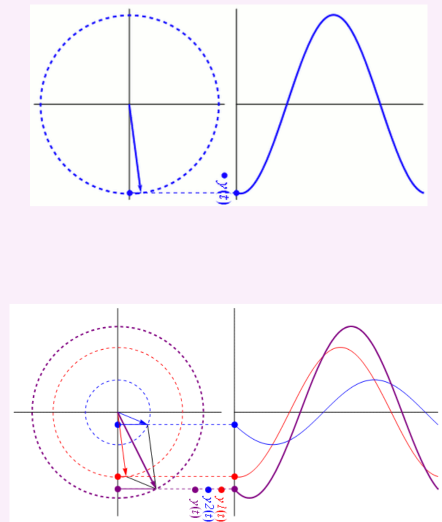
2



Sinusoidal signal with same frequency and amplitude but different phase



Phasor representation



Rotating phasor

9.3 What are Phasors (1)

Phasors provide a simple means of analyzing linear circuits excited by sinusoidal sources.

A phasor is a complex number that represents the amplitude and phase of a sinusoidal function.

We have 3 representation for complex number z

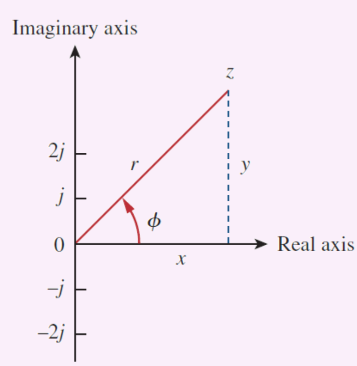
$$z = x + jy$$ Rectangular form

Where, $j = \sqrt{-1}$, x is the real part of z and y is the imaginary part of z

$$z = r\angle\phi$$ Polar form

Where, r is the magnitude of z and ϕ is the phase of z

$$z = re^{j\phi}$$ Exponential form



9.3 What are Phasors (2)

Relationship between rectangular form and polar form

Rectangular ⇒ polar

$$r = \sqrt{x^2 + y^2} \quad \& \quad \phi = \tan^{-1}\frac{y}{x}$$

polar ⇒ Rectangular

$$x = r\cos\phi \quad \& \quad y = r\sin\phi$$

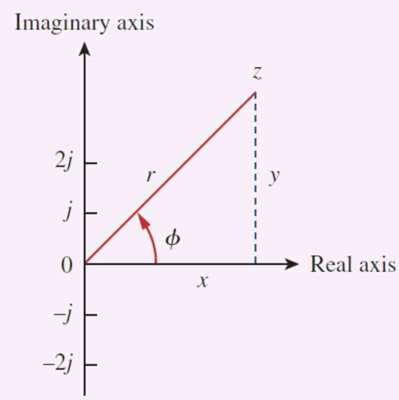
since, $z = x + jy$

$$z = r\cos\phi + jr\sin\phi$$

$$z = r(\cos\phi + j\sin\phi)$$

but, $e^{j\phi} = \cos\phi + j\sin\phi$

$$z = re^{j\phi}$$



9.3 Phasors Operations

Basic properties of complex numbers

Addition: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$

Subtraction: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$

Multiplication: $z_1 z_2 = r_1 r_2 \underline{/\phi_1 + \phi_2}$

Division: $\frac{z_1}{z_2} = \frac{r_1}{r_2} \underline{/\phi_1 - \phi_2}$

Reciprocal: $\frac{1}{z} = \frac{1}{r} \underline{/-\phi}$

Square Root: $\sqrt{z} = \sqrt{r} \underline{/\phi/2}$

Complex Conjugate: $z^* = x - jy = r\underline{/-\phi} = re^{-j\phi}$

Remember

$$z = x + jy$$
$$z = r\angle\phi$$
$$z = re^{j\phi}$$

$$r = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\frac{y}{x}$$

Euler's identity

$$e^{j\phi} = \cos\phi + j\sin\phi$$
$$\cos\phi = \mathrm{Re}(e^{j\phi})$$
$$\sin\phi = \mathrm{Im}(e^{j\phi})$$

$$j = \sqrt{-1}$$
$$j^2 = -1$$
$$\frac{1}{j} = -j$$

9.3 Phasors format of a sinusoidal function

The idea of phasor representation is based on Euler's identity

$$e^{j\phi} = \cos\phi + j\sin\phi$$

$$\mathrm{Re}(e^{j\phi}) = \cos\phi \quad \text{and} \quad \mathrm{Im}(e^{j\phi}) = \sin\phi$$

Let the sinusoidal function is

$$v(t) = V_m \cos(\omega t + \phi)$$

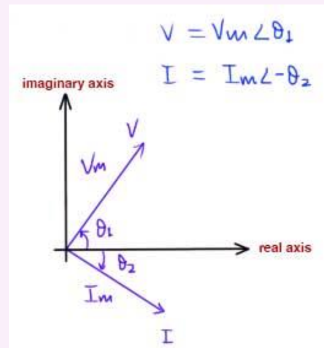
$$v(t) = \mathrm{Re}(V_m e^{j(\omega t+\phi)})$$

$$v(t) = \mathrm{Re}(V_m e^{j\phi} e^{j\omega t})$$

$$v(t) = \mathrm{Re}(\boldsymbol{V} e^{j\omega t})$$

Where **V** is the *phasor representation (magnitude* V_m *and phase* ϕ*)* of the sinusoid $v(t)$

$$\boldsymbol{V} = V_m e^{j\phi} = V_m \angle\phi$$



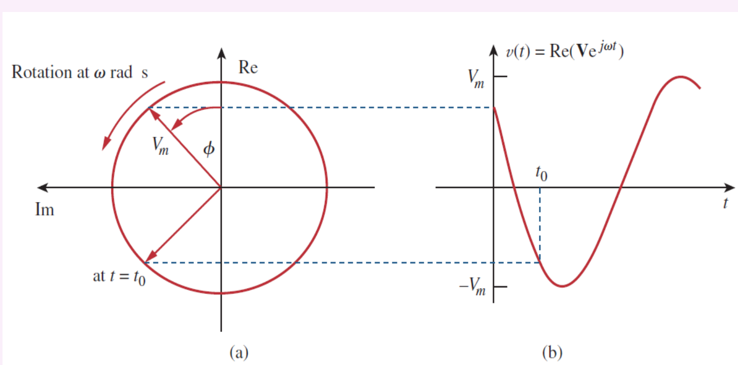
Dr. Hazem Falah Sakeek | www.hazemsakeek.net | www.physicsacademy.org

7

7

9.3 Phasors equivalence

Representation of $V e^{j\omega t} = V_m\, e^{j(\omega t+\phi)}$



Dr. Hazem Falah Sakeek | www.hazemsakeek.net | www.physicsacademy.org

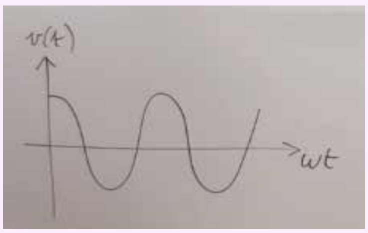
8

8

9.3 Difference between time domain and phasor domain

Time domain representation

$v(t) = V_m \cos(\omega t + \phi)$

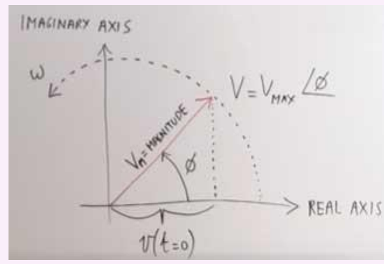


Tells us the voltage as a function of time

Tells us the angular frequency

Phasor domain representation

$\mathbf{V} = V_m \underline{/\phi}$



Snapshot at t=0

Tells us the magnitude and phase angle

9

9

9.3 Phasors of derivative and integral

From Eqs. $v(t) = \mathrm{Re}(\boldsymbol{V} e^{j\omega t}) \;\; = V_m \cos(\omega t + \phi)$

So that

$$\frac{dv}{dt} = -\omega V_m \sin(\omega t + \phi) \quad = \omega V_m \cos(\omega t + \phi + 90^o)$$

$$= \mathrm{Re}(\omega V_m e^{j\omega t} e^{j\phi} e^{j90}) \; = \mathrm{Re}(j\omega \boldsymbol{V} e^{j\omega t})$$

This shows that the derivative of $v(t)$ is transformed to the phasor domain as $j\omega\boldsymbol{V}$

$$\frac{dv}{dt} \quad \Leftrightarrow \quad j\omega\mathbf{V}$$

(Time domain) (Phasor domain)

The integral of $v(t)$ is transformed to the phasor domain as $\boldsymbol{V}/j\omega$

$$\int v\, dt \quad \Leftrightarrow \quad \frac{\mathbf{V}}{j\omega}$$

(Time domain) (Phasor domain)

10

10

9.3 Phasors

The differences between $v(t)$ and $\boldsymbol{V}$ should be emphasized:

1. $v(t)$ is the *instantaneous or time domain* representation, while **V** is the *frequency or phasor domain* representation.
2. $v(t)$ is time dependent, while **V** is not.
3. $v(t)$ is always real with no complex term, while **V** is generally complex.

Note that: phasor analysis applies only when frequency is constant; it applies in manipulating two or more sinusoidal signals only if they are of the same frequency.

Example

Let $v_1 = 50\angle 60^o$ and $v_2 = 30\angle -50^o$ calculate $v_1 + v_2$

Solution:

$$v_1 = 50\angle 60^o = 50(\cos 60 + j\sin 60) = 25 + j\,43.3$$

$$v_2 = 30\angle -50^o = 30(\cos(-50) + j\sin(-50)) = 19.3 - j\,23$$

$$v_1 + v_2 = 44.3 + j\,20.3$$

In polar format

$$V_M = \sqrt{44.3^2 + 20.3^2} = 48.7$$

$$\phi = \tan^{-1}\frac{20.3}{44.3} = 24.6^o$$

$$v_1 + v_2 = 48.7\angle 24.6^o$$

Example

Evaluate the complex numbers $\dfrac{20\angle -30^o + (4-j3)}{(4+j9)(5-j2)}$

Solution:

$$\frac{20(\cos -30 + j\sin -30) + (4-j3)}{20 - j\,8 + j\,45 - j^2\,18} \qquad \text{but } j^2 = -1$$

$$= \frac{17.3 - j\,10 + (4-j3)}{38 - j37} = \frac{21.3 - j\,13}{38 - j37}$$

$$= \frac{\sqrt{21.3^2 + 13^2}\angle \tan^{-1}\dfrac{-13}{21.3}}{\sqrt{38^2 + 37^2}\angle \tan^{-1}\dfrac{-37}{38}} \qquad = \frac{24.95\angle -31.4^o}{53.04\angle -44.2^o}$$

$$= 0.47\angle -31.4 - (-44.2) \qquad = 0.47\angle 12.8^o$$

Dr. Hazem Falah Sakeek | www.hazemsakeek.net | www.physicsacademy.org

13

13

Example (1)

Let $v_1 = -10\sin(\omega t - 30^o)$
and $v_2 = 20\cos(\omega t + 45^o)$, calculate $v_1 + v_2$

Solution:

We need to convert v_1 to the form $v = +V_m\cos(\omega t + \phi)$
First convert the –ve sign to +ve by adding 180°

$$v_1 = +10\sin(\omega t - 30^o + 180^o) \quad = +10\sin(\omega t + 150^o)$$

Second convert the sine function to cosine

$$v_1 = +10\cos(\omega t + 150^o - 90^o) = +10\cos(\omega t + 60^o)$$

Write the phasor format of v_1 and v_2

$$v_1 = 10\angle 60^o \quad = 10(\cos 60 + j\sin 60) \quad = 5 + j\,8.66$$

$$v_2 = 20\angle 45^o \quad = 20(\cos 45 + j\sin 45) \quad = 14.14 + j\,14.14$$

$$v_1 + v_2 = 19.14 + j\,22.8$$

Dr. Hazem Falah Sakeek | www.hazemsakeek.net | www.physicsacademy.org

14

14

Example (2)

$$v_1 + v_2 = 19.14 + j\,22.8$$

Convert the answer from phasor domain to time domain

$$v_1 + v_2 = V_M \cos(\omega t + \phi)$$

$$V_M = \sqrt{19.14^2 + 22.8^2} = 29.8$$

$$\phi = \tan^{-1}\frac{22.8}{19.14} = 50^o$$

$$v_1 + v_2 = 29.8\cos(\omega t + 50^o)$$

Example 9.7

Using the phasor approach, determine the current $i(t)$

$$4i + 8\int i dt - 3\frac{di}{dt} = 50\cos(2t + 75^o)$$

Transform each term in the equation from time domain to phasor domain.

$$4I + \frac{8I}{\omega j} - 3I\omega j = 50\angle 75^o$$

But $\omega = 2$

$$4I + \frac{4I}{j} - j6I = 50\angle 75^o$$

But $\frac{1}{j} = -j$

$$4I - j4I - j6I = 50\angle 75^o$$

$$\mathrm{I}(4 - j10) = 50\angle 75^o$$

Convert $4 - j10$ to polar form

Example 9.7

$4 - j10 \quad \rightarrow \quad r\angle\phi$

$r = \sqrt{4^2 + 10^2} = 10.77$

$\phi = \tan^{-1}\frac{-10}{4} = -68.2^o$

$\mathrm{I}(4 - j10) = 50\angle 75^o$

$\mathrm{I}(10.77\angle -68.2^o) = 50\angle 75^o$

$$\mathrm{I} = \frac{50\angle 75^o}{10.77\angle -68.2^o}$$

$\mathrm{I} = 4.64\angle 143.2^o \quad \Rightarrow \quad i(t) = 4.64\cos(2t + 143.2^o)$

Practice Problem 9.7

Find the voltage $v(t)$ in a circuit using the phasor approach.

$$2\frac{dv}{dt} + 5v + 10\int v\,dt = 50\cos(5t - 30^o)$$

Transform each term in the equation from time domain to phasor domain.

$$2V\,j\omega + 5V + \frac{10V}{j\omega} = 50\angle -30^o$$

But $\omega = 5$ and $\frac{1}{j} = -j$

$j10V + 5V - j2V = 50\angle -30^o$

$V(5 + j8) = 50\angle -30^o$

$V(9.43\angle 58^o) = 50\angle -30^o$

$$V = \frac{50\angle -30^o}{9.43\angle 58^o} = 5.3\angle -88^o$$

$$\therefore v(t) = 5.3\cos(5t - 88^o)$$

$(5 + j8) \quad \rightarrow \quad r\angle\phi$

$r = \sqrt{5^2 + 8^2} = 9.43$

$\phi = \tan^{-1}\frac{8}{5} = 58^o$

Problems to Solve by yourself

(1) Calculate these complex numbers and express your results in rectangular form:

(a) $\dfrac{60\angle 45^\circ}{7.5 - j10} + j2$

(b) $\dfrac{32\angle -20^\circ}{(6 - j8)(4 + j2)} + \dfrac{20}{-10 + j24}$

(c) $20 + (16\angle -50^\circ)(5 + j12)$

(2) Find the phasors corresponding to the following signals:

(a) $v(t) = 21\cos(4t - 15^\circ)$ V

(b) $i(t) = -8\sin(10t + 70^\circ)$ mA

(c) $v(t) = 120\sin(10t - 50^\circ)$ V

(d) $i(t) = -60\cos(30t + 10^\circ)$ mA

(3)

Two voltages v_1 and v_2 appear in series so that their sum is $v = v_1 + v_2$. If $v_1 = 10\cos(50t - \pi/3)$ V and $v_2 = 12\cos(50t + 30^\circ)$ V, find v.

19