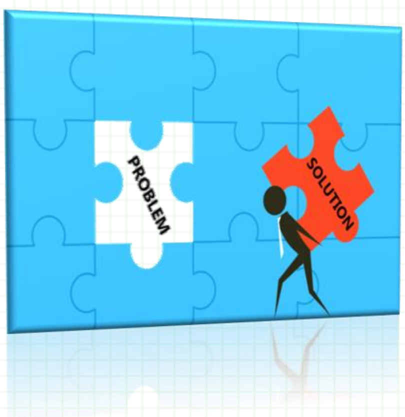
Physics Academy

www.physicsacademy.org

Electric Circuits



Unit: 6 | Lecture: 30

Solution of some selected problem

Capacitors and Inductors

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

1

1

Problems 6.1

If the voltage across a 7.5-F capacitor is $2te^{-3t}$ V find the current and the power.

The current

$$i = C\frac{dv}{dt}$$

$$= 7.5(2e^{-3t} - 6te^{-3t})$$

$$= 15(1 - 3t)e^{-3t} \text{ A}$$

The power

$$p = vi$$

$$= 2te^{-3t} \times 15(1 - 3t)e^{-3t}$$

$$= 30t(1 - 3t)e^{-6t} \text{ W}$$

2

2

Problems 6.4

A current of $4\,sin\,4t\;A$ flows through a 5-F capacitor.
Find the voltage $v(t)$ across the capacitor given that $v(0) = 1$ V.

The voltage $v(t)$

$$v(t) = \frac{1}{C}\int_{t_o}^{t} i(\tau)\,d\tau + v(t_o)$$

$$= \frac{1}{5}\int_{0}^{t} 4\,sin\,4t\,d\tau + 1$$

$$= \frac{4}{5}\left(\frac{-\cos 4t}{4}\right)\Bigg|_0^t + 1$$

$$= -0.2\cos 4t\Big|_0^t + 1$$

$$= -0.2\cos 4t - (-0.2\cos 0) + 1$$

$$= -0.2\cos 4t + 0.2 + 1$$

$$= 1.2 - 0.2\cos 4t \quad \text{V}$$

Dr. Hazem Falah Sakeek | www.hazemsakeek.net | www.physicsacademy.org

3

Problems 6.12

A voltage of $30e^{-2000t}$ V appears across a parallel combination of a 100-mF capacitor and a 12Ω resistor. Calculate the power absorbed by the parallel combination.

The power $p = vi$

We have the voltage, so we need to find the current through the resistor i_R and the current in the capacitor i_C.

$$i_R = \frac{v}{R} = \frac{30e^{-2000t}}{12} = 2.5\,e^{-2000t}\text{A}$$

$$i_C = C\frac{dv}{dt} = 100\times10^{-3}\frac{d30e^{-2000t}}{dt}$$

$$= 0.1\times30\times(-2000)\,e^{-2000t} = -6000\,e^{-2000t}\ \text{A}$$

The current $i = i_R + i_C = -5997.5\,e^{-2000t}$ A

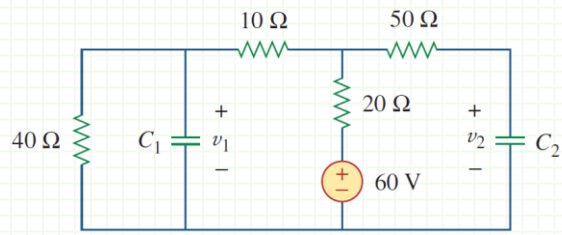
The power $p = vi = -5997.5\,e^{-2000t}\times30e^{-2000t} = -180\,e^{-4000t}$ W

Dr. Hazem Falah Sakeek | www.hazemsakeek.net | www.physicsacademy.org

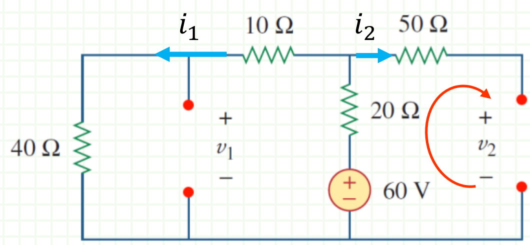
4

Problems 6.13

Find the voltage across the capacitors in the circuit under dc conditions.



Under dc conditions, the circuit is as shown



$$i_1 = \frac{60}{40 + 10 + 20} = 0.857 \text{ A}$$

$$i_2 = 0$$

$$v_1 = 40 \times i_1 = 34.3 \text{ V}$$

Apply KVL to find v_2

$$v_2 - 60 + 20 i_1 = 0$$

$$v_2 = 42.86 \text{ V}$$

5

5

Problems 6.14

Series-connected 20-pF and 60-pF capacitors are placed in parallel with series-connected 30-pF and 70-pF capacitors. Determine the equivalent capacitance.

20 pF is in series with 60pF $= \dfrac{20 \times 60}{20 + 60} = 15 \text{ pF}$

30-pF is in series with 70pF $= \dfrac{30 \times 70}{30 + 70} = 21 \text{ pF}$

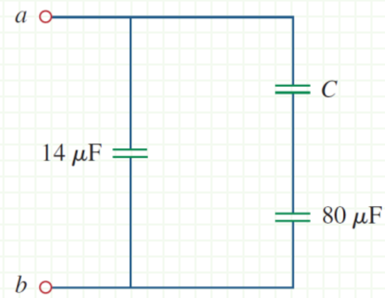
15pF is in parallel with 21pF $= 15 + 21 = 36 \text{ pF}$

6

6

Problems 6.16

The equivalent capacitance at terminals *a-b* in the circuit is 30 μF. Calculate the value of *C*.



$$C_{eq} = 14 + \frac{C \times 80}{C + 80} = 30$$

$$\frac{C \times 80}{C + 80} = 16$$

$$80C = 16C + 1280$$

$$64C = 1280$$

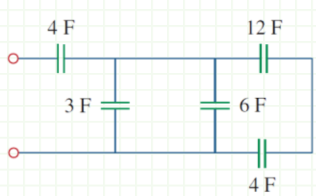
$$C = 20\ \mu F$$

7

7

Problems 6.17

Determine the equivalent capacitance for each of the circuits.



4F in series with 12F

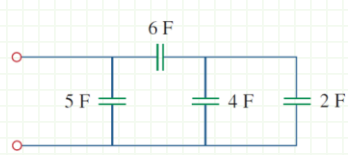
$$= \frac{4 \times 12}{4 + 12} = 3\ \text{F}$$

3F in parallel with 6F and 3F

$$= 3 + 6 + 3 = 12\ \text{F}$$

4F in series with 12F

$$C_{eq} = \frac{4 \times 12}{4 + 12} = 3\ \text{F}$$



2F in parallel with 4F

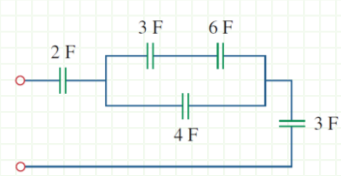
$$= 2 + 4 = 6\ \text{F}$$

6F in series with 6F

$$= \frac{6 \times 6}{6 + 6} = 3\ \text{F}$$

3F in parallel with 5F

$$C_{eq} = 3 + 5 = 8\ \text{F}$$



3F in series with 6F

$$= \frac{3 \times 6}{3 + 6} = 2\ \text{F}$$

2F in parallel with 4F

$$= 2 + 4 = 6\ \text{F}$$

6F, 2F, and 3F are in series

$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{2} + \frac{1}{3} = 1$$

$$C_{eq} = 1\text{F}$$

8

8

Problems 6.34

The current through a 10-mH inductor is $10e^{-t/2}$ A. Find the voltage and the power at t=3s.

The voltage

$$v = L\frac{di}{dt}$$

$$v = 10 \times 10^{-3}\frac{d\left(10e^{-t/2}\right)}{dt}$$

$$= 10 \times 10^{-3} \times 10 \times \frac{-1}{2}e^{-t/2}$$

$$= -50 \times 10^{-3}e^{-t/2}$$

$$v(3) = -50 \times 10^{-3}e^{-3/2}$$

$$= -0.011157 \text{ V}$$

The power

$$p = vi$$

$$= -50 \times 10^{-3}e^{-t/2} \times 10e^{-t/2}$$

$$= -500 \times 10^{-3}e^{-t}$$

$$p(3) = -500 \times 10^{-3}e^{-3}$$

$$p(3) = -0.025 \text{ W}$$

Problems 6.35

An inductor has a linear change in current from 50 mA to 100 mA in 2 ms and induces a voltage of 160 mV. Calculate the value of the inductor.

$$v = L\frac{di}{dt}$$

$$L = \frac{v}{\frac{di}{dt}}$$

$$L = \frac{160 \times 10^{-3}}{\frac{(100-50)10^{-3}}{2 \times 10^{-3}}}$$

$$L = 6.4 \times 10^{-3} \text{ H}$$

Problems 6.39

The voltage across a 200-mH inductor is given by $v(t) = 3t^2 + 2t + 4$ V for t>0. Determine the current i(t) through the inductor. Assume that i(0)= 1A.

$$i = \frac{1}{L}\int_{t_o}^{t} v(\tau)\, d\tau \; + i(t_o)$$

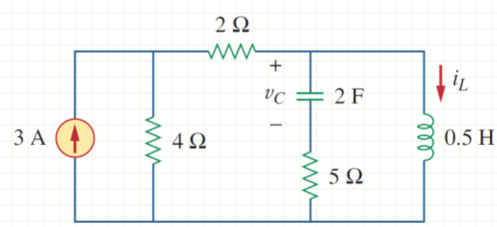
$$i = \frac{1}{200 \times 10^{-3}} \int_0^t 3t^2 + 2t + 4\, d\tau + i(0)$$

$$i = 5\; t^3 + t^2 + 4t\Big|_0^t + 1$$

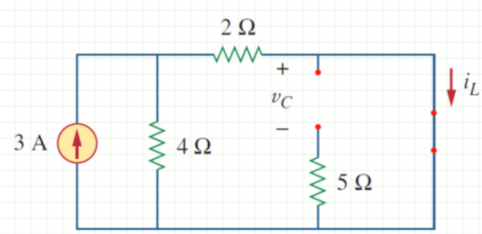
$$i(t) = 5t^3 + 5t^2 + 20t + 1 \quad \text{A}$$

Problems 6.46

Find v_C, i_L and the energy stored in the capacitor and inductor in the circuit under dc conditions.



Under dc conditions, the circuit is as shown



Using current division to find i_L

$$i_L = \frac{4}{4+2} \times 3 = 2 \text{ A}$$

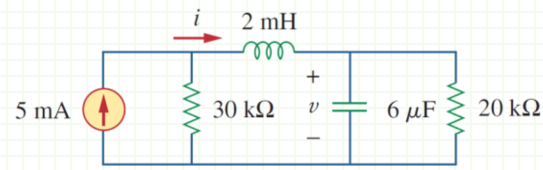
$$v_C = 0 \text{ V}$$

$$W_L = \frac{1}{2} L\, i_L^2 \;\; = \frac{1}{2} \times 0.5 \times 2^2 = 1 \text{ J}$$

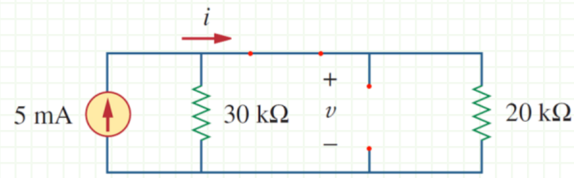
$$W_C = \frac{1}{2} C\, v_C^2 \;\; = 0 \text{ J}$$

Problems 6.48

Under steady-state dc conditions, find i and v in the circuit.



Under dc conditions, the circuit is as shown



Using current division to find i

$$i = \frac{30}{30+20} \times 5 = 3 \text{ mA}$$

$$v = 20 \times i \quad = 20 \times 3 = 60 \text{ V}$$

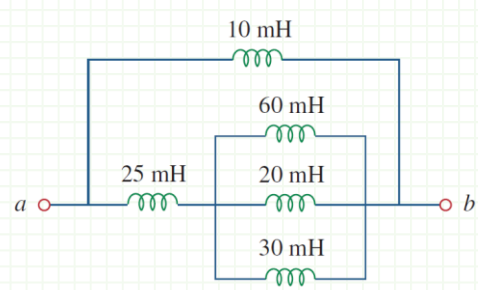
Dr. Hazem Falah Sakeek | www.hazemsakeek.net | www.physicsacademy.org

(13)

13

Problems 6.51

Determine L_{eq} at terminals a-b of the circuit



$$\frac{1}{L} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30}$$

$$\frac{1}{L} = \frac{1}{10}$$

$$L = 10$$

$$L_{eq} = 10||(25 + 10)$$

$$L_{eq} = \frac{10 \times 35}{10 + 35} = 7.778 \text{ mH}$$

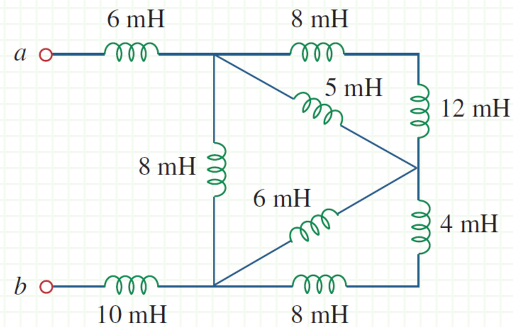
Dr. Hazem Falah Sakeek | www.hazemsakeek.net | www.physicsacademy.org

(14)

14

Problems 6.53

Find L_{eq} at the terminals of the circuit



5 mH is in parallel with 8 mH and 12 mH

$$= 5||(8+12)$$

$$= \frac{5\times(8+12)}{5+(8+12)} \quad = 4\ \text{mH}$$

6 mH is in parallel with 4 mH and 8 mH

$$= 6||(4+8)$$

$$= \frac{6\times(4+`8)}{6+(4+8)} \quad = 4\ \text{mH}$$

8 mH is in parallel with 4 mH and 4 mH

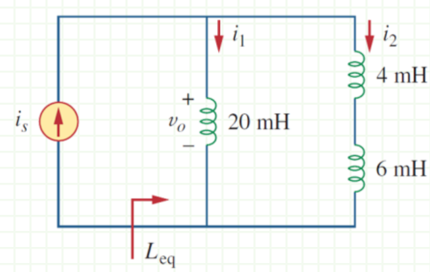
$$= 8||(4+4) \quad = 4\ \text{mH}$$

6 mH, 10 mH and 4 mH are in series

$$L_{eq} = 6+10+4 = 20\ \text{mH}$$

Problems 6.61

Consider the circuit. Find: (a) L_{eq} and $i_1(t)$, $i_2(t)$ if $i_s = 3e^{-t}\text{mA}$ (b) $v_o(t)$, (c) energy stored in the 20-mH inductor at t=1 s.



(a) L_{eq}

$$L_{eq} = 20||(4+6) \; = \frac{20\times 10}{20+10} = 6.667\ \text{mH}$$

Using current division to find i_1(t) and i_2(t)

$$i_1(t) = \frac{10}{10+20}\times i_s \; = \frac{1}{3}\times 3e^{-t} = e^{-t}\ \text{mA}$$

$$i_2(t) = \frac{20}{10+20}\times i_s \; = \frac{2}{3}\times 3e^{-t} = 2e^{-t}\ \text{mA}$$

(b) $v_o(t)$

$$v_o = L_{eq}\frac{di_s}{dt}$$

$$= 6.667\times 10^{-3}(-3\times 10^{-3}\times e^{-t})$$

$$-20\times 10^{-6}\ e^{-t}\ \text{V}$$

(c) energy stored in the 20-mH inductor at t=1 s.

$$W = \frac{1}{2}Li_1^2$$

$$= \frac{1}{2}\times 20\times 10^{-3}\times e^{-2}\times 10^{-6}$$

$$= 1.353\times 10^{-9}\ \text{J}$$