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Electric Circuits

Unit: 6 | Lecture: 29

Capacitors and Inductors:
Series and Parallel Inductors

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Capacitors and Inductors

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- 6.2 Capacitors
- 6.3 Series and Parallel Capacitors
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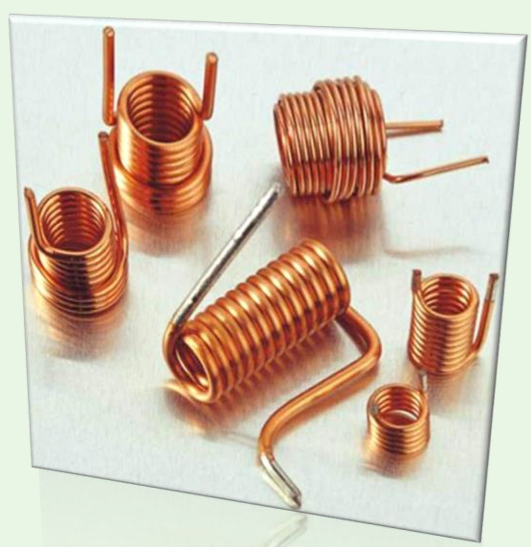
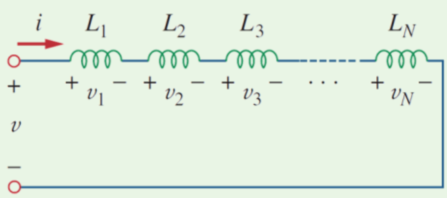
- 6.5 Series and Parallel Inductors



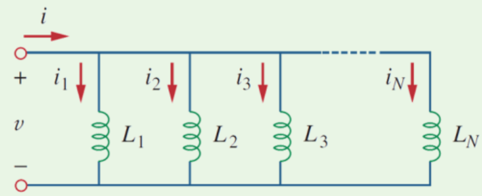
6.5 Series and Parallel Inductors (1)



Series-connected inductors

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

The equivalent inductance of series-connected inductors is the sum of the individual inductances.



Parallel -connected inductors

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

The equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

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Relation	**Resistor (R)**	**Capacitor (C)**	**Inductor (L)**
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit

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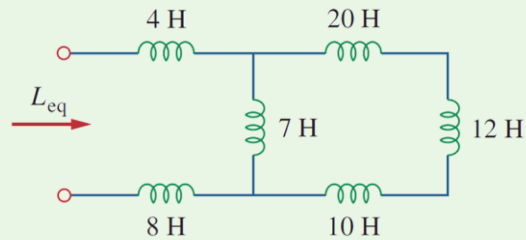
Example 6.11

Find the equivalent inductance of the circuit



The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H

This 42-H inductor is in parallel with the 7-H inductor

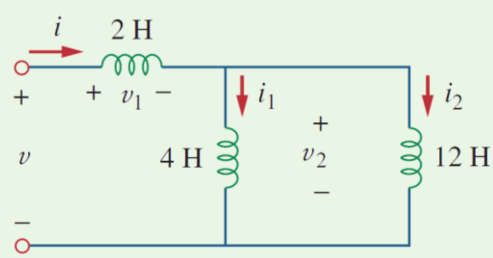
$$\frac{7 \times 42}{7 + 42} = 6 \text{ H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors

$$L_{eq} = 4 + 6 + 8 = 18 \text{ H}$$

Example 6.12

For the circuit $i(t) = 4(2 - e^{-10t})\text{mA}$. If $i_2(0) = -1 \text{ mA}$. Find (a) $i_1(0)$, (b) v(t), $v_1(t)$, $v_2(t)$, (c) $i_1(t)$ and $i_2(t)$.



(a) $i(t) = 4(2 - e^{-10t})\text{mA}$

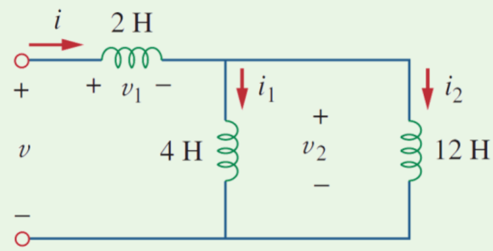
Note that $i = i_1 + i_2$

$$i(0) = 4(2 - 1) = 4 \text{ mA}$$

$$i_1(0) = i(0) - i_2(0) \quad = 4 - (-1) = 5 \text{ mA}$$

Example 6.12

For the circuit $i(t) = 4(2 - e^{-10t})\text{mA}$. If $i_2(0) = -1\text{ mA}$. Find (a) $i_1(0)$, (b) $v(t)$, $v_1(t)$, $v_2(t)$, (c) $i_1(t)$ and $i_2(t)$.



(b) The equivalent inductance is

$$L_{eq} = 2 + 4 \parallel 12 = 2 + 3 = 5\text{ H}$$

$$v(t) = L_{eq}\frac{di}{dt} = 5(4)(-1)(-10)e^{-10t}\text{ mV} = 200e^{-10t}\text{ mV}$$

$$v_1(t) = 2\frac{di}{dt} = 2(-4)(-10)e^{-10t}\text{ mV} = 80e^{-10t}\text{ mV}$$

$$v_2(t) = v(t) - v_1(t) = 120e^{-10t}\text{ mV}$$

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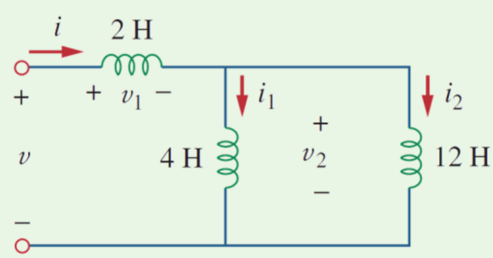
Example 6.12

For the circuit $i(t) = 4(2 - e^{-10t})\text{mA}$. If $i_2(0) = -1\text{ mA}$. Find (a) $i_1(0)$, (b) $v(t)$, $v_1(t)$, $v_2(t)$, (c) $i_1(t)$ and $i_2(t)$.



(c) The current i_1 is obtained as

$$i_1(t) = \frac{1}{4}\int_0^t v_2\,dt + i_1(0) = \frac{120}{4}\int_0^t e^{-10t}\,dt + 5\text{ mA}$$

$$= -3e^{-10t}\Big|_0^t + 5\text{ mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t}\text{ mA}$$

$$i_2(t) = \frac{1}{12}\int_0^t v_2\,dt + i_2(0) = \frac{120}{12}\int_0^t e^{-10t}\,dt - 1\text{ mA}$$

$$= -e^{-10t}\Big|_0^t - 1\text{ mA} = -e^{-10t} + 1 - 1 = -e^{-10t}\text{ mA}$$

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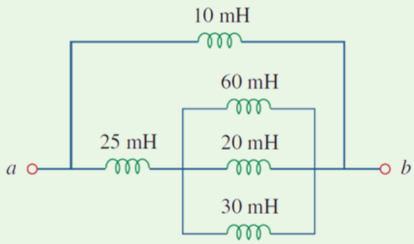
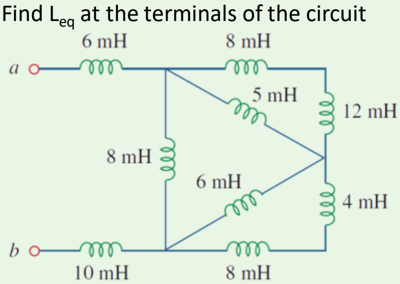
Problems to Solve by yourself

(1) Determine at terminals *a-b* of the circuit



(2) Find L_{eq} at the terminals of the circuit



(3) Consider the circuit. Find: (a) L_{eq} and $i_1(t)$, $i_2(t)$ if $i_s = 3e^{-t}$mA (b) $v_o(t)$, (c) energy stored in the 20-mH inductor at *t*=1 s.