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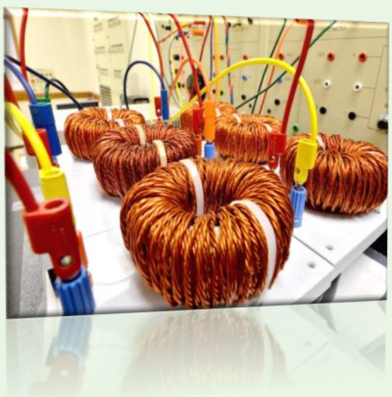
Electric Circuits



Unit: 6 | Lecture: 28

Capacitors and Inductors: Inductors

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Capacitors and Inductors

6.1 Introduction
6.2 Capacitors
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6.5 Series and Parallel Inductors

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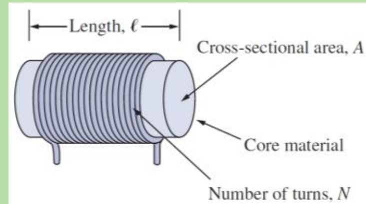
6.4 Inductors (1)

An inductor is a passive element designed to store energy in its magnetic field.

Inductors are used in power supplies, transformers, radios, TVs, radars, and electric motors.

Any conductor of electric current has inductive properties. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire.

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).



An inductor consists of a coil of conducting wire.

$$v = L\frac{di}{dt}$$

where L is the constant of proportionality called the *inductance* of the inductor. The unit of inductance is the henry (H),

1 henry equals 1 volt-second per ampere.

$$L = \frac{N^2 \mu A}{l}$$ L for solenoid

where N is the number of turns, l is the length, A is the cross-sectional area, and μ is the permeability of the core.

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6.4 Inductors (2)

The current-voltage relationship is obtained from Eq.

$$v = L\frac{di}{dt}$$

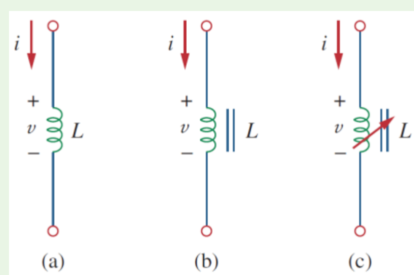
$$\therefore di = \frac{1}{L}\, v\, dt$$

Integrating gives

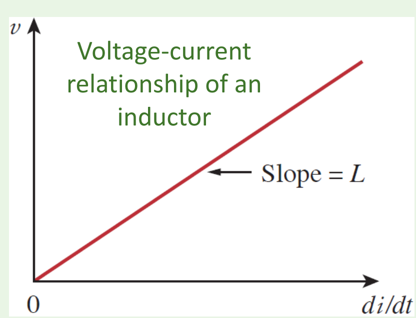
$$i = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\, d\tau$$

$$i = \frac{1}{L}\int_{t_o}^{t} v(\tau)\, d\tau + i(t_o)$$

Where $i(t_o)$ is the total current for $-\infty < t < t_o$ and $i(-\infty) = 0$.



(a) air-core, (b) iron-core, (c) variable iron-core.



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6.4 Inductors (3)

The inductor is designed to store energy in its magnetic field.

The power delivered to the inductor is

$$p = vi = \left(L\frac{di}{dt}\right)i$$

The energy stored in the inductor is therefore

$$w = \int_{-\infty}^{t} p(\tau)d\tau \quad = L\int_{-\infty}^{t} \frac{di}{d\tau}\, i d\tau \quad = L\int_{-\infty}^{v(t)} i\, di \quad = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

Since $i(-\infty) = 0$

$$w = \frac{1}{2}Li^2$$

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6.4 Inductors (4)

Important properties of an inductor.

The voltage across an inductor is zero when the current is constant. Thus, An inductor acts like a short circuit to dc.

An important property of the inductor is its opposition to the change in current flowing through it. The current through an inductor cannot change instantaneously

Ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.

A practical, nonideal inductor has a significant resistive component.

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Example 6.8

The current through a 0.1-H inductor is $i(t) = 10t\, e^{-5t} A$. Find the voltage across the inductor and the energy stored in it.

$$v = L\frac{di}{dt} \qquad L = 0.1\text{ H}$$

$$v = 0.1\frac{d}{dt}(10t\, e^{-5t}) \quad = e^{-5t} + t(-5)e^{-5t} \quad = e^{-5t}(1 - 5t)\text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 \quad = \frac{1}{2}(0.1)100t^2e^{-10t} \quad = 5t^2e^{-10t}\text{ J}$$

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Example 6.9

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

$$\because i = \frac{1}{L}\int_{t_o}^{t} v(\tau)\, d\tau + i(t_o) \qquad L = 5\text{ H}$$

$$i = \frac{1}{5}\int_0^t 30t^2\, dt + 0 \quad = 6 \times \frac{t^3}{3} = 2t^3\text{ A}$$

The energy stored

$$w\Big|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li^2(0) \quad = \frac{1}{2}(5)(2 \times 5^3)^2 - 0 \quad = 156.25\text{ kJ}$$

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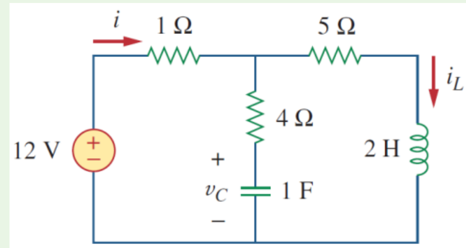
Example 6.10

Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.

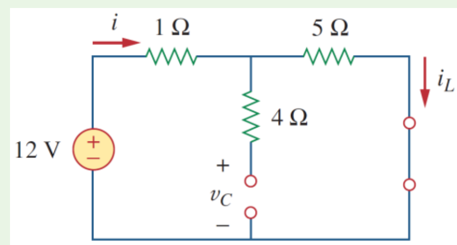


$$i = i_L = \frac{12}{1 + 5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5Ω

$$v_C = 5i = 10 \text{ V}$$

Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit



The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

The energy in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

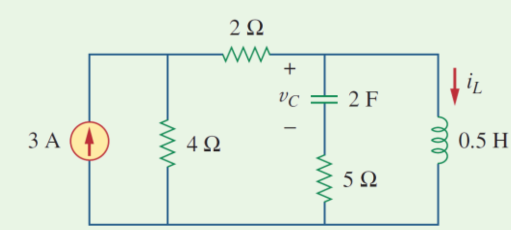
Problems to Solve by yourself

(1) The current through a 10-mH inductor is $10e^{-t/2}$ A. Find the voltage and the power at t=3s.

(2) An inductor has a linear change in current from 50 mA to 100 mA in 2 ms and induces a voltage of 160 mV. Calculate the value of the inductor.

(3) The voltage across a 200-mH inductor is given by $v(t) = 3t^2 + 2t + 4$ V for t>0. Determine the current i(t) through the inductor. Assume that i(0)= 1A.

(4) Find v_C, i_L and the energy stored in the capacitor and inductor in the circuit under dc conditions.



(5) Under steady-state dc conditions, find i and v in the circuit.