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Electric Circuits

Unit: 3 | Lecture: 15

Solution of some selected problem

Nodal analysis & Mesh analysis

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Problems 3.2 Obtain the node voltages in the circuit.

Draw the current direction at each node

Apply KCL at node 1,

$$\frac{0-v_1}{10}+\frac{0-v_1}{5}=6+\frac{v_1-v_2}{2}$$

$$-8v_1+5v_2=60 \qquad (1)$$

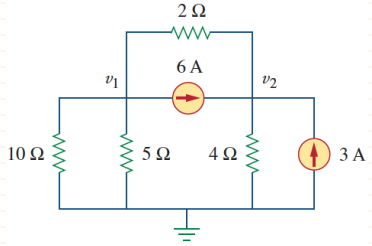
Apply KCL at node 2,

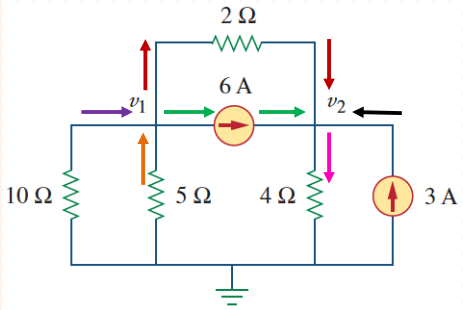
$$3+6+\frac{v_1-v_2}{2}=\frac{v_2-0}{4}$$

$$-2v_1+3v_2=36 \qquad (2)$$

Solve equations (1) & (2)

$$v_1=0 \quad \& \quad v_2=12$$





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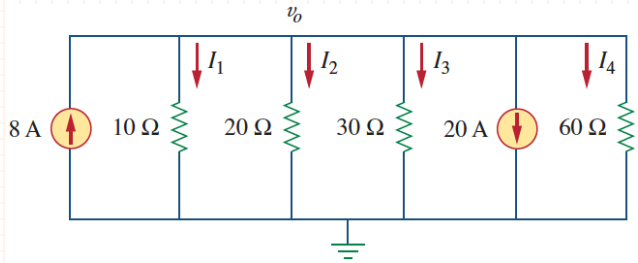
Problems 3.3

Find the currents I_1 through I_4 and the voltage v_o in the circuit.



We have one nonreference node

Apply KCL

$$I_1 + I_2 + I_3 + 20 + I_4 = 8$$

$$\frac{v_0}{10} + \frac{v_0}{20} + \frac{v_0}{30} + 20 + \frac{v_0}{60} = 8$$

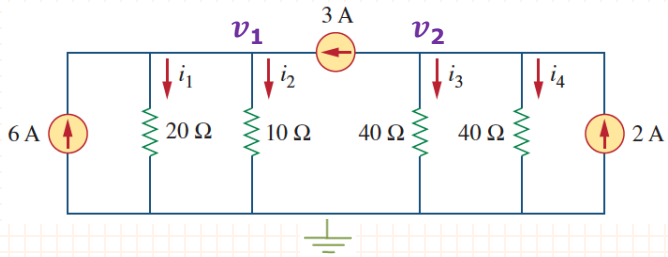
$$v_0 = -60 \text{ V}$$

$$I_1 = \frac{v_0}{10} = -6\ A \qquad I_2 = \frac{v_0}{20} = -3\ A$$

$$I_3 = \frac{v_0}{30} = -2\ A \qquad I_4 = \frac{v_0}{60} = -1\ A$$

Problems 3.4

Find the currents I_1 through I_4 and the voltage v_o in the circuit.



We have two nonreference node around the current source 3 A.

Apply KCL at node 1

$$6 + 3 = i_1 + i_2$$

$$6 + 3 = \frac{v_1}{20} + \frac{v_1}{10}$$

$$v_1 = 60 \text{ V}$$

Apply KCL at node 2

$$2 = 3 + i_3 + i_4$$

$$2 = 3 + \frac{v_2}{40} + \frac{v_2}{40}$$

$$v_2 = -20 \text{ V}$$

$$i_1 = \frac{v_1}{20} = \frac{60}{20} = 3\ A$$

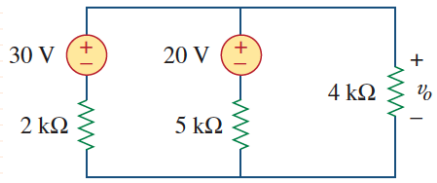
$$i_2 = 6\ A$$

$$i_3 = -0.5\ A$$

$$i_4 = -0.5\ A$$

Problems 3.5 (1)

Obtain v_o in the circuit

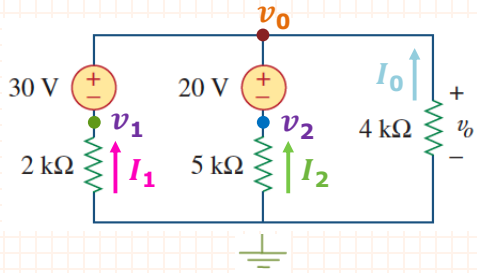


Apply KCL at node v_o

$$I_1 + I_2 + I_0 = 0$$

but

$$v_0 = -4000 I_0 \quad \Rightarrow \quad I_0 = -\frac{v_0}{4000}$$



Notice that

$$v_0 - v_1 = 30$$

$$v_0 + 2000 I_1 = 30$$

$$\therefore I_1 = \frac{30 - v_0}{2000}$$

$$v_0 - v_2 = 20$$

$$v_0 + 5000 I_2 = 20$$

$$\therefore I_2 = \frac{20 - v_0}{5000}$$

Problems 3.5 (2)

$$I_1 + I_2 + I_0 = 0$$

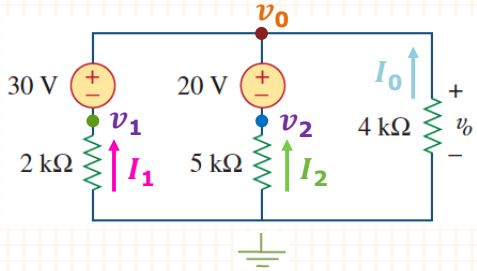
$$\frac{30 - v_0}{2} + \frac{20 - v_0}{5} - \frac{v_0}{4} = 0$$

$$\frac{10(30 - v_0)}{20} + \frac{4(20 - v_0)}{20} - \frac{5v_0}{20} = 0$$

$$300 - 10v_0 + 80 - 4v_0 - 5v_0 = 0$$

$$380 - 19v_0 = 0$$

$$v_0 = 20\ V$$



$$I_0 = -\frac{v_0}{4000}$$

$$I_1 = \frac{30 - v_0}{2000}$$

$$I_2 = \frac{20 - v_0}{5000}$$

Problems 2.7

Apply nodal analysis to solve for V_x in the circuit.

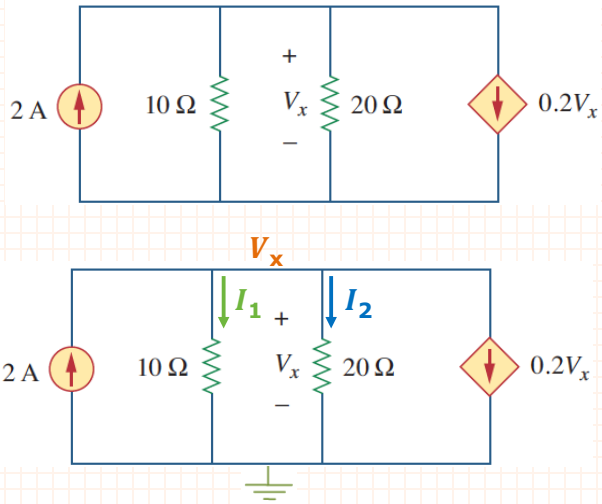
Applying KCL to the circuit

$$2 = I_1 + I_2 + 0.2V_x$$

$$2 = \frac{V_x - 0}{10} + \frac{V_x - 0}{20} + 0.2V_x$$

$$40 = 2V_x + V_x + 4V_x$$

$$V_x = \frac{40}{7} = 5.714\ V$$



Problems 2.8

Using nodal analysis, find v_o in the circuit.

Applying KCL to the circuit

$$I_1 + I_2 + I_3 = 0$$

$$\frac{v_1 - 0}{6+4} + \frac{(v_1 - 60) - 0}{20} + \frac{(v_1 - 5v_0) - 0}{20} = 0$$

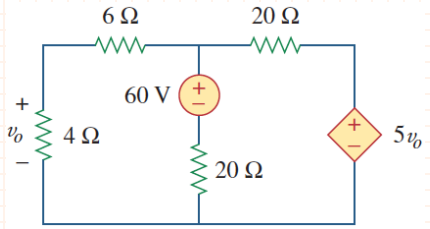
$$\frac{v_1}{10} + \frac{v_1 - 60}{20} + \frac{v_1 - 5v_0}{20} = 0$$

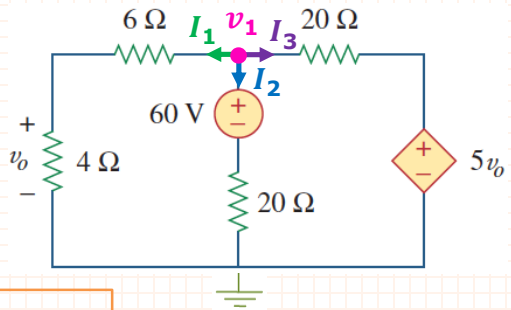
$$4v_1 - 5v_0 = 60$$

Using voltage divider to replace v_o

$$v_0 = \frac{4}{4+6} v_1 \quad \rightarrow \quad v_1 = \frac{10}{4} v_0$$

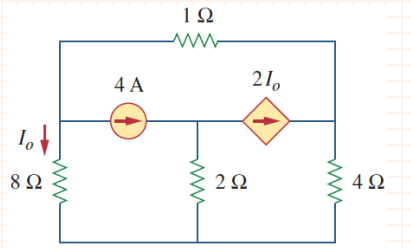
$$4 \times \frac{10}{4} v_0 - 5v_0 = 60 \quad \rightarrow \quad v_0 = \frac{60}{5} = 12\ V$$





Problems 2.10

Find I_o in the circuit.



At node 1 $\frac{v_1}{8} + 4 + \frac{v_1 - v_3}{1} = 0$ → $9v_1 - 8v_3 = -32$ (1)

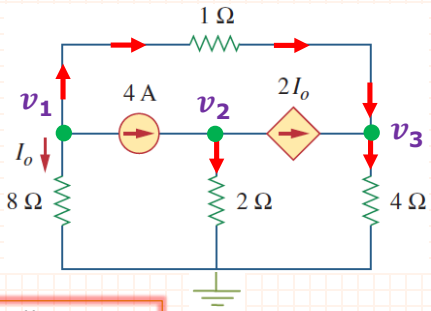
At node 2 $\frac{v_2 - 0}{2} + 2I_0 = 4$ But $I_0 = \frac{v_1}{8}$

$\frac{v_2}{2} + \frac{2v_1}{8} = 4$ → $v_1 + 2v_2 = 16$ (2)

At node 3 $\frac{v_1 - v_3}{1} + 2I_0 = \frac{v_3}{4}$

$v_1 - v_3 + \frac{2v_1}{8} = \frac{v_3}{4}$ → $5v_1 - 5v_3 = 0$

$\therefore v_1 = v_3$ (3)

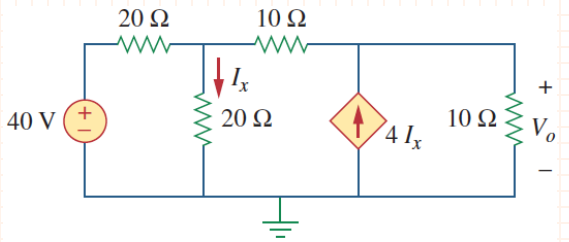


Substitute eqn. (3) in eqn. (1)

$9v_1 - 8v_1 = -32$ → $v_1 = -32\ V$ → $\therefore I_0 = \frac{v_1}{8} = -4\ A$

Problems 2.12

Using nodal analysis, determine V_o in the circuit.



At node 1 $\frac{v_1 - 40}{20} + \frac{v_1}{20} + \frac{v_1 - v_2}{10} = 0$

$v_1 - 40 + v_1 + 2v_1 - 2v_2 = 0$

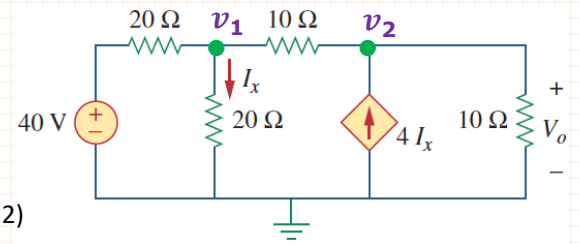
$4v_1 - 2v_2 = 40$ (1)

At node 2 $\frac{v_1 - v_2}{10} + 4I_x = \frac{v_2}{10}$

But $I_x = \frac{v_1}{20}$

$v_1 - v_2 - 2v_1 = v_2$

$3v_1 = 2v_2$ → $v_1 = 0.666v_2$ (2)



Substitute eqn. (2) in eqn. (1) $4 \times 0.666v_2 - 2v_2 = 40$ → $V_0 = v_2 = 60\ V$